Definition 2.1 A subset $A$ of a topological space $(X, \tau)$ is called

1) generalized closed set (briefly $g$-closed) [5] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$.
2) semi-generalized closed set (briefly $sg$-closed) [2] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi-open in $(X, \tau)$.
3) $\psi g$-closed set [10] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $sg$-open in $(X, \tau)$.
4) $\psi g$-closed set [8] if $\psi \text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$.
5) $\psi \alpha$-closed set [1] if $\psi \text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\psi g$-open in $(X, \tau)$.
6) The closure operator of $\psi \alpha$-closed set is defined as $\psi \alpha \text{cl}(A) = \cap \{F \subseteq X: A \subseteq F \text{ and } F \text{ is } \psi \alpha$-closed in $(X, \tau)\}$ [1]

Definition 2.2 A topological space $(X, \tau)$ is said to be a

(i) $T_{\psi}$-space if every g-$\psi$-closed subset of $(X, \tau)$ is closed in $(X, \tau)$. [4]
(ii) $T_{\alpha}$-space if every g-$\alpha$-closed subset of $(X, \tau)$ is g-$\alpha$-closed in $(X, \tau)$. [4]
(iii) $T_{\psi}$-space if every g-$\psi$-closed subset of $(X, \tau)$ is g-$\psi$-closed in $(X, \tau)$. [9]
(iv) $T_{\psi g}$-space if every g-$\psi g$-closed subset of $(X, \tau)$ is g-$\psi g$-closed in $(X, \tau)$. [9]
(v) $T_{\psi g}$-space if every g-$\psi$-closed subset of $(X, \tau)$ is g-$\psi$-closed in $(X, \tau)$. [6]
(vi) $T_{\alpha}$-space if every g-$\psi$-closed subset of $(X, \tau)$ is g-$\alpha$-closed in $(X, \tau)$. [6]
(vii) $T_{\psi g}$-space if every g-$\psi g$-closed subset of $(X, \tau)$ is g-$\alpha$-closed in $(X, \tau)$. [6]
(viii) $T_{\alpha}$-space if every g-$\psi g$-closed subset of $(X, \tau)$ is g-$\alpha$-closed in $(X, \tau)$. [9]
(ix) $T_{\psi}$-space if every g-$\psi$-closed subset of $(X, \tau)$ is g-$\psi$-closed in $(X, \tau)$. [9]
(x) $\alpha$-space if every g-$\alpha$-closed subset of $(X, \tau)$ is closed in $(X, \tau)$. [7]
(xi) $\psi$-space if every $\psi$-closed subset of $(X, \tau)$ is closed in $(X, \tau)$. [10]
**Proposition 3.11** Theorem 4.5 [1]. Therefore \( x \) is closed in \((X, \tau)\).

**Proposition 3.2** Let \( \tau \) be a \( \psi \alpha \)-space. Then \( x \) is closed in \((X, \tau)\).

**Example 3.3** Let \( \tau = \{a, b, c\} \) with topology \( \tau = \{\emptyset, \{a\}, \{a, b, c\}\} \).

**Theorem 3.7** Let \( A \) be a closed set in \((X, \tau)\). Hence \((X, \tau)\) is \( \psi \alpha \)-closed.

**Theorem 3.6** Let \( A \) be a \( \psi \alpha \)-closed set in \((X, \tau)\). Hence \((X, \tau)\) is \( \psi \alpha \)-closed.

**Theorem 3.5** Let \( A \) be a \( \psi \alpha \)-closed set in \((X, \tau)\). Hence \((X, \tau)\) is \( \psi \alpha \)-closed.

**Theorem 3.4** Let \( A \) be a \( \psi \alpha \)-closed set in \((X, \tau)\). Hence \((X, \tau)\) is \( \psi \alpha \)-closed.

**Theorem 3.3** Let \( A \) be a \( \psi \alpha \)-closed set in \((X, \tau)\). Hence \((X, \tau)\) is \( \psi \alpha \)-closed.

**Theorem 3.2** Let \( A \) be a \( \psi \alpha \)-closed set in \((X, \tau)\). Hence \((X, \tau)\) is \( \psi \alpha \)-closed.

**Theorem 3.1** Let \( A \) be a \( \psi \alpha \)-closed set in \((X, \tau)\). Hence \((X, \tau)\) is \( \psi \alpha \)-closed.

**Example 3.1** Let \( A \) be a \( \psi \alpha \)-closed set in \((X, \tau)\). Hence \((X, \tau)\) is \( \psi \alpha \)-closed.
Example 3.12 Let \( X = \{a, b, c\} \) with topology \( \tau = \{\phi, [a], [b, c], X\} \). Then \((X, \tau)\) is a \( \psi \alpha T_a \) space but not a \( T_b \) space, since the subsets \([b], [c], [a, b]\) and \([a, c]\) are \( g_\alpha \)-closed but not closed in \((X, \tau)\).  

Proposition 3.13 Every \( aT_b \)-space is a \( \psi \alpha T_a \)-space but not conversely.  

Proof: Let \((X, \tau)\) be a \( aT_b \)-space and let \( A \) be a \( \psi \alpha \)-closed set in \((X, \tau)\). By proposition 3.10(1) \( A \) is \( g_\alpha \)-closed in \((X, \tau)\). Since \((X, \tau)\) is a \( aT_b \)-space, \( A \) is closed in \((X, \tau)\) and so it is \( \alpha \)-closed in \((X, \tau)\). Hence \((X, \tau)\) is \( \psi \alpha T_a \)-space.  

Example 3.14 Let \( X = \{a, b, c\} \) with topology \( \tau = \{\phi, [a], X\} \). Then \((X, \tau)\) is a \( \psi \alpha T_a \) space but not a \( aT_b \) space, since the subsets \([a, b]\) and \([a, c]\) are \( \alpha \)-closed but not closed in \((X, \tau)\).  

Proposition 3.15 Every \( \psi \alpha \) space is a \( \psi \alpha T_a \) space but not conversely.  

Proof: Let \((X, \tau)\) be a \( \psi \alpha \) space and let \( A \) be a \( \psi \alpha \)-closed set in \((X, \tau)\). By proposition 3.24(1) \( A \) is \( \psi \alpha \)-closed in \((X, \tau)\). Since \((X, \tau)\) is a \( \psi \alpha \) space, \( A \) is closed in \((X, \tau)\) and so it is \( \alpha \)-closed in \((X, \tau)\). Hence \((X, \tau)\) is \( \psi \alpha T_a \)-space.  

Example 3.16 Let \( X = \{a, b, c\} \) with topology \( \tau = \{\phi, [a], X\} \). Then \((X, \tau)\) is a \( \psi \alpha T_a \) space but not a \( \psi \alpha \) space, since the subsets \([b]\) and \([c]\) are \( \psi \alpha \)-closed but not closed in \((X, \tau)\).  

Remark 3.17 The following examples show that \( \psi \alpha T_a \)-space is independent from \( g_\alpha T_{\psi \alpha \tau} \)-space, \( \alpha T_a \)-space and \( \psi \alpha T_{\psi \alpha \tau} \)-space  

Example 3.18 Let \( X = \{a, b, c\} \) with topology \( \tau = \{\phi, [a], [b, c], X\} \). Then \((X, \tau)\) is a \( \psi \alpha T_a \) space but not a \( g_\alpha T_{\psi \alpha \tau} \) space and not a \( \psi \alpha T_{\psi \alpha \tau} \) space, since the subsets \([b, c]\), \([a, b]\) and \([a, c]\) are \( g_\alpha \)-closed, \( \alpha \)-closed and \( \psi \alpha \) closed in \((X, \tau)\).  

Example 3.19 Let \( X = \{a, b, c\} \) with topology \( \tau = \{\phi, [a], X\} \). Then \((X, \tau)\) is a \( g_\alpha T_{\psi \alpha \tau} \)-space, \( \alpha T_a \)-space and \( \psi \alpha T_{\psi \alpha \tau} \)-space but not a \( \psi \alpha T_a \) space, since the subsets \([a, c]\) and \([b, c]\) are \( \psi \alpha \)-closed but not \( \alpha \)-closed in \((X, \tau)\).  

Remark 3.20 The space \( \psi \alpha T_a \) is independent of \( \alpha T_a \) space and \( \psi \alpha \) space as seen from the following example.  

Example 3.21 Let \( X = \{a, b, c\} \) with topology \( \tau = \{\phi, [a], [b, c], X\} \). Then \((X, \tau)\) is an \( \alpha \)-space, \( \alpha T_a \) space and \( \alpha T_a \) space, not \( \psi \alpha T_a \) space, since the subsets \([a, c]\) and \([b, c]\) are \( \psi \alpha \)-closed but not \( \alpha \)-closed in \((X, \tau)\).  

Example 3.22 Let \( X = \{a, b, c\} \) with topology \( \tau = \{\phi, [a], [a, b], X\} \). Then \((X, \tau)\) is a \( \psi \alpha T_a \) space but not an \( \alpha \)-space, \( \alpha T_a \) space and \( \psi \alpha \) space, since the subset \([b]\) is \( \alpha \)-closed and \( \alpha g_\alpha \)-closed but not closed, \( \psi \alpha \)-closed and \( \psi \alpha \)-closed in \((X, \tau)\).  

Remark 3.23 The following examples show that \( \psi \alpha T_a \) space and \( T_{1/2} \) space are independent.  

Example 3.24 Let \( X = \{a, b, c\} \) with topology \( \tau = \{\phi, [a], [b, c], X\} \). Then \((X, \tau)\) is a \( T_{1/2} \) space but not a \( \psi \alpha T_a \) space, since the subsets \([a, c]\) and \([b, c]\) are \( \psi \alpha \)-closed but not \( \alpha \)-closed in \((X, \tau)\).  

Example 3.25 Let \( X = \{a, b, c\} \) with topology \( \tau = \{\phi, [a], X\} \). Then \((X, \tau)\) is a \( \psi \alpha T_a \) space but not a \( T_{1/2} \) space, since the subsets \([b]\), \([c]\), \([a, b]\) and \([a, c]\) are \( g_\alpha \)-closed but not \( g_\alpha \)-closed in \((X, \tau)\).  

Proposition 3.26 Every \( g_\alpha T_{\psi \alpha \tau} \) space is a \( g_\alpha T_{\psi \alpha \tau} \) space but not conversely.  

Proof: The proof follows from the fact that every \( g_\alpha \)-closed set is \( g_\alpha \)-closed.  

Example 3.27 Let \( X = \{a, b, c\} \) with topology \( \tau = \{\phi, [a], [a, b], X\} \). Then \((X, \tau)\) is a \( g_\alpha T_{\psi \alpha \tau} \) space but not \( g_\alpha T_{\psi \alpha \tau} \) space, since the subset \([a, c]\) is \( g_\alpha \)-closed but not \( \psi \alpha \)-closed in \((X, \tau)\).  

Proposition 3.28 Every \( \psi \alpha T_{\psi \alpha \tau} \) space is a \( g_\alpha T_{\psi \alpha \tau} \) space and \( \alpha T_a \)-space but not conversely.  

Proof: The proof follows from the fact that every \( g_\alpha \)-closed set and \( g_\alpha \)-closed set is \( \psi \alpha \)-closed.  

Example 3.29 Let \( X = \{a, b, c\} \) with topology \( \tau = \{\phi, [a], [b, a], X\} \). Then \((X, \tau)\) is a \( g_\alpha T_{\psi \alpha \tau} \) space and a \( g_\alpha T_{\psi \alpha \tau} \) space but not a \( \psi \alpha T_{\psi \alpha \tau} \) space, since the subsets \([a]\) and \([b]\) are \( \psi \alpha \)-closed but not \( \psi \alpha \)-closed in \((X, \tau)\).  

Remark 3.30 The spaces \( g_\alpha T_{\psi \alpha \tau} \) space, \( \alpha T_a \)-space and \( \psi \alpha T_{\psi \alpha \tau} \) space are independent with \( \psi \alpha \) space.  

Example 3.31 Let \( X = \{a, b, c\} \) with topology \( \tau = \{\phi, [a], [b, c], X\} \). Then \((X, \tau)\) is a \( \psi \alpha \) space but not a \( g_\alpha T_{\psi \alpha \tau} \) space, not a \( \psi \alpha T_{\psi \alpha \tau} \) space and not a \( \psi \alpha T_{\psi \alpha \tau} \) space, since the subsets \([b]\), \([c]\), \([a, b]\) and \([a, c]\) are \( g_\alpha \)-closed, \( \alpha \)-closed and \( \psi \alpha \) closed but not \( \psi \alpha \)-closed in \((X, \tau)\).  

Example 3.32 Let \( X = \{a, b, c\} \) with topology \( \tau = \{\phi, [a, b], X\} \). Then \((X, \tau)\) is a \( g_\alpha T_{\psi \alpha \tau} \) space, a \( \alpha T_a \)-space and a \( \psi \alpha T_{\psi \alpha \tau} \) space but not \( \psi \alpha \) space, since the subsets \([a, c]\) and \([b, c]\) are \( \psi \alpha \)-closed but not closed in \((X, \tau)\).
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