ABSTRACT
Let $G = \{V(G), E(G)\}$ be a simple graph and $f: v \mapsto \{1, 2, \ldots, |V|\}$ be a bijection. For each edge $uv$, assign the label 1. If either $[f(u)]^2 / f(v)$ or $[f(v)]^2 / f(u)$ and the label 0 otherwise $f$ is called a square divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph with a square divisor cordial labeling is called a square divisor cordial graph. In this work, a discussion is made on Shell, Tensor product, Coconut tree, Jelly fish and Subdivision of bistar under square divisor cordial labeling.

KEYWORDS: SDCL, Tensor Product, Coconut Tree, Jelly Fish, Shell graph.

DEFINITIONS

Tensor Product:
The Tensor product $G_1 \otimes G_2$ of two simple graphs $G_1$ and $G_2$ is the graph with $V(G_1 \otimes G_2) = V_1V_2$. Where $(u_1, u_2)$ and $(v_1, v_2)$ are adjacent in $G_1$, $G_2$ if and only if $u_1$ is adjacent to $v_1$ in $G_1$ and $u_1$ is adjacent to $v_2$ in $G_2$. The Tensor product of $K_{1,n}$ and $P_2$.

Coconut Tree:
A Coconut Tree $CT(m, n)$ is the graph obtained from the path $P_m$ by appending $n$ new pendent edges at an end vertex of $P_m$.

Jelly Fish:
The Jelly fish graph $J(m, n)$ is obtained by joining a 4-cycle whose vertices are $v_1, v_2, v_3, v_4$ with vertices $v_1$ and $v_3$ defined by an edge and appending $m$ pendent edges to $v_2$ and $n$ pendent edges to $v_4$.

Subdivision of a graph:
$A$ is a graph that can be obtained from $G$ by a sequence of edge subdivisions are called Subdivision of a graph. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions.
**Shell graph:**
A shell $S_n$ is the graph obtained by taking $n - 3$ concurrent chords in a cycle $C_n$ on $n$ vertices. The vertex at which all the chords are concurrent is called the apex vertex. The shell is also called a fan $F_{n-1}$, i.e., $S_n = F_{n-1} = F_{n-1} + K_1$.

**RESULTS**

**Theorem:**
The shell graph $S_n$ is a Square Divisor Cordial Graph.

**Proof:**
Let $G = S_n$ be the shell graph. By the definition of shell graph the order and size of $G$ are $p = n + 1$ and $q = 2n - 1$. Define the vertex set by $V = \{x, u_1, u_2, \cdots, u_n\}$, where $x$ be an apex vertex, $u_i$'s are adjacent to the apex vertex and $u_i$'s are connected by successive vertices. Define the edge set $E$ as $E = E_1 \cup E_2$, where $E_1 = e_i = (x, u_i)$ and $E_2 = e_{ii} = \{(u_1, u_2), (u_2, u_3), \ldots, (u_{n-1}, u_n)\}$, where $i \in N$. We define the labeling $f : V(S_n) \to \{1, 2, 3, \ldots, n + 1\}$, where $f(x) = 1$; $f(u_i) = i + 1, \forall 1 \leq i \leq n$.

Therefore, the shell graph $S_n$ is a Square Divisor Cordial graph.

**Example:**

Square Divisor Cordial labeling of the Shell graph $S_6$ is shown in the Fig 3.1.

$|5 - 6| \leq 1$.

Hence, the given Shell graph $S_6$ is Square Divisor Cordial Graph.

**Theorem:**
The Tensor Product graph $(G_1 \ (Tp) \ G_2)$ is a Square Divisor Cordial Graph.

**Proof:**
Let $G = G_1(T_p)G_2$ be a tensor product graph of order $p = 2(n + 1)$ and size $= 2n$ . Let the vertex $V = \{u, u_1, u_2, \ldots, u_n, v, v_1, v_2, \ldots, v_n\}$, where $u, v$ are the apex vertices, $u_i$'s are adjacent and pendent vertices of $u$ and $v_i$'s are adjacent and pendent vertices of $v$. The edge set $E$ is defined as $= \{e_1, e_2\}$, where $e_1 = e_i = (u, u_i)$ and $e_2 = e_{ii} = (v, v_i) \ \forall \ i = 1, 2, \ldots, n$. The bijective function $f : V(G) \to \{1, 2, 3, \cdots, 2(n + 1)\}$ is defined as,

$f(u) = 1$; $f(v) = 2$;

$f(u_i) = 2i + 2, \forall 1 \leq i \leq n$;

$f(v_i) = 2i + 1, \forall 1 \leq i \leq n$;

Such that $e_f(0) = n$ and $e_f(1) = n$.

Therefore, by the definition of Square Divisor Cordial Graph, $|e_f(0) - e_f(1)| \leq 1$.

$|n - n| \leq 1$.

Hence, the Tensor product graph $G_1(T_p)G_2$ is a Square Divisor Cordial Graph.

**Example:**
Square Divisor Cordial labeling of Tensor Product $(G_1(T_p)G_2)$ shown in Fig 3.2.
Fig 3.2 SDCL of Tensor Product

In this Fig 3.2, \( e_f(0) = 4 \) and \( e_f(1) = 4 \).

By the definition of Square Divisor Cordial Labeling,
\[
| e_f(0) - e_f(1) | \leq 1 ,
\[
| 4 - 4 | \leq 1 .
\]

Hence, the Tensor product graph \( G_1(T_p)G_2 \) has a Square Divisor Cordial Labeling.

**Theorem:**
The Coconut Tree \( CT(n, n - 2) \) is a Square Divisor Cordial Graph.

**Proof:**
Let \( CT(n, n - 2) \) be the Coconut Tree. The order of \( CT(n, n - 2) \) is \( p = n + n - 2 \) and size \( = n + n - 3 \).

By the definition of Coconut Tree, the vertex set \( (G) = \{u_1, u_2, ..., u_n, v_1, v_2, ..., v_{n-2}\} \).

Let \( u_1, u_2, ..., u_n \) be the vertices of the path \( p_n \) and \( v_1, v_2, ..., v_{n-2} \) be the pendent vertices attached by the beginning vertex of path \( p_n \).

The edge set \( E(G) = \{e_{ni}, e_{ij}\} \), where \( e_{1i} = (u_i, v_i) \) and \( e_{ij} = (u_i, u_j) \) \( \forall \ i, j = 1, 2, ..., n \)

Let us define the function \( f : V(G) \rightarrow \{1, 2, ..., n + n - 2\} ; \)
\[
f(u_i) = i, \forall 1 \leq i \leq n ;
\]
\[
f(v_i) = q + 1 ;
\]
Such that \( e_f(0) = n - 2 \) and \( e_f(1) = n - 1 , \)
\[
| n - 2 - n + 1 | \leq 1 ;
\]
\[
| e_f(0) - e_f(1) | \leq 1 .
\]

Which is condition of the Square Divisor Cordial Labeling. Thus the Coconut Tree \( CT(n, n - 2) \) is a Square Divisor Cordial Graph.

**Example:**
Square Divisor Cordial Labeling of \( CT(5, 3) \) shown in Fig. 3.3.
**Theorem:**
The graph Jelly Fish $J(m, m)$ is Square Divisor Cordial Graph.

**Proof:**
Let $G = J(m, m)$ of order $p = 2m + 4$ and size $q = 2m + 5$ be a jelly fish. The vertex set and edge set of $f(m, m)$ are

$V(f(m, m)) = \{(u,v,x,y), [u,v, y \forall 1 \leq i \leq m]\}$

$E(f(m, m)) = \{(ux) \cup (uy) \cup (vx) \cup (vy) \cup (xy) \cup [(uv), \forall 1 \leq i \leq m] \}$

Let us define the function $f : V(G) \rightarrow \{1, 2, ..., 2m + 4\}$ as follows:

$f(u) = 1$; $f(v) = 2$;

$f(x) = 2m + 3$;

$f(y) = 2m + 4$;

$f(u_i) = 2i + 2, \forall 1 \leq i \leq n$;

$f(v_i) = 2i + 1, \forall 1 \leq i \leq n$. The above function induces $e_f(0) = m + 2$ and $e_f(1) = m + 3$

$\lvert |m + 2 - (m + 3)\rvert \leq 1$.

Hence, the graph Jelly Fish $J(m, m)$ is a Square Divisor Cordial Graph.

**Example:**
The Jelly Fish $J(4, 4)$ is shown in Fig 3.4.

$e_f(0) = 6$ and $e_f(1) = 7$.

$\lvert |e_f(0) - e_f(1)| \leq 1$.

$\lvert 6 - 7\rvert \leq 1$.

Fig 3.4 SDCL of $J(4, 4)$

Hence, the graph Jelly Fish $J(4, 4)$ is a Square Divisor Cordial Labeling.

**Theorem:**
The graph obtained by the Subdivision of the central edge of the bistar $(B_{n,n})$ has a Square Divisor Cordial Graph.

**Proof:**
Let $G$ be the graph obtained by the Subdivision of the central edge of the bistar $B_{n,n}$. Let the vertex set and edge set of the graph $B_{2n}$are

$V(G) = \{u,v,w,u_i,v_i\}, \forall 1 \leq i \leq n$;

$E(G) = \{uw,wv,u_i,v_i\}, \forall 1 \leq i \leq n$;

The order of $G$ is $p = 2n + 3$ and the size $q = 2n + 2$. Let us define the function $f : V(G) \rightarrow \{1, 2, \cdots, 2n + 3\}$ by

$f(u) = 1$; $f(v) = 2$; $f(w) = 3$;

$f(u_i) = 2i + 2, \forall 1 \leq i \leq n$;

$f(v_i) = 2i + 3, \forall 1 \leq i \leq n$;

$e_f(0) = n + 1$ and $e_f(1) = n + 1$.

Therefore by the definition of Square Divisor Cordial Graph, $\lvert e_f(0) - e_f(1)\rvert \leq 1$.

Hence, the Subdivision of the central edge of the bistar $(B_{n,n})$ is a Square Divisor Cordial Graph.

**Example:**
The Square Divisor Cordial Labeling of Subdivision $B_{4,4}$ is shown in Fig 3.5.
Fig 3.5 SDCL of $B_{4,4}$

$e_f(0) = 5$ and $e_f(1) = 5$.

$|e_f(0) - e_f(1)| \leq 1.$

Hence, the Subdivision of graph $B_{4,4}$ is a Square Divisor Cordial Labeling.

CONCLUSION
The overview of Square Divisor Cordial Graphs is the current interest due to its diversified applications. Here we investigate some results corresponding to labeled graphs. Similar work can be carried out for the other graphs also. The complied information related to SDCL will be useful for researchers to get some idea related to their field.

REFERENCES