ABSTRACT

Diabetes Mellitus (DM) is a chronic disease characterized by a deficiency in insulin production and its action or both. There are a number of herbal medicines have been recommended for the treatment of Diabetes. The traditional plant medicines were used throughout the world for a range of Diabetic representations. Recently, Mathematical and Computational techniques have been becoming a united with the Biology and Medicine for consume the computational time. In the present work, three selected medicinal plants i.e. the Helicteres isora Linn. (H. isora), the Portulaca oleracea L. (P. oleracea) and the Caralluma attenuata (C. attenuata) for Diabetes models of physical significance in two nonlinear differential equations considered and their exact and analytical solutions are obtained. The activity of three plants was confirmed by these techniques and it is found to be safety, efficiency and accuracy in Diabetic. Thus, the obtained results from these techniques are a very good agreement with the laboratory experimental results.

KEYWORDS: Diabetes; Medicinal Plants; Herbal Medicine; Analytical Solution.

I. INTRODUCTION

Diabetes is chronic pathology which is characterized by hyper glycaemia caused by defective insulin action, insulin secretion, or the combination of both. Diabetes and its complications continue to be a major problem in the world population [1]. Currently, available synthetic drugs for treating this disease are found to be associated with many adverse effects. However, plants and plant-derived products have proven to be effective and safe in the treatment of various types of diabetes mellitus. The use of plants in medicine is an ancient practice in different parts of the world for the preventive and curative purposes [2]. In natural system of medicine, several medicinal plants have found potential use as the blood sugar lowering agents. Experimentally induced diabetes in animals has provided significant depth into the biochemical and physiological derangement of the diabetic state. Many of them have been scientifically explored for their usefulness in managing diabetes [3].

Numerous and varied reports abound in literature on studies conducted to investigate the significant effects of medicinal plants on various pathophysiological complications of diabetes mellitus. In the present study, three following medicinal plants were considered on the basis of their previously published work. Helicteres isora Linn. (H. isora) is found to be significant in diabetes. Cumulative research findings on root extract of H. isora showed various anti-hyperglycemic and hypolipidemic study signifying the potential use of the extract in the treatment of type-2 diabetes [4-6].

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Portulaca oleracea L., (P. oleracea) is commonly known as Ma-Chi-Xian in Chinese. This plant possesses a wide spectrum of pharmacological properties like antidiabetic, antioxidant, anti-antitumorogenic and anticancer activities [7]. Its seeds and polysaccharides have been reported for significant antidiabetic potential in streptozotocin (STZ)-induced hyperglycemia and oxidative stress [8, 9]. Caralluma attenuata (C. attenuata) also known as ‘Kundaietikommu’ indigenously. This plant is known for its antihyperglycemic activity [10]. Ethnopharmacological claims support for its use in gastric ulcers and diabetes [11].

Mathematical techniques has been united with the art and individuality of biology and medicine, providing indispensable tools for the design [12,13]. Considering this view about the comprehensive mathematical analysis of selected signify their efficacy data in the treatment of Diabetes Mellitus. In our previous study confirmed the above selected medicinal plants for their anti-Diabetic response with help of statistical and mathematical tools [14].

Mathematical Models in Biology and Medicine were new theory to developing the computational effort with numerical techniques [15] as well as approximate analytical methods [16]. The importance of analytical tool for nonlinear problems, namely HAM, is employed to solve ordinary differential equations (7)-(8) with initial conditions (5).

The analytical and numerical techniques has been successfully applied to many nonlinear problems in Bio-Maths such as biochemical reaction model of fractional order [17], Model of Diabetic population [18], Insulin-Glucose feedback and regulatory system of Diabetes [19,20], Diabetes Compartment Modelling [21], detecting Diabetes in the Blood [22], and Fractional modelling dynamics of HIV and CD4+T-cells [23]. The biological resources model Homoclinic orbit equations [24] and predator prey system [25] studied on various special cases of the differential equations. All of these successful applications of the analytical and numerical techniques verify its validity for nonlinear problems in science.

The present study based on $\left( \frac{c}{\tau} \right)$–expansion method [26-28] and homotopy analysis method (HAM) [29-30], which are exact and analytical solutions for nonlinear ordinary differential equations. The results obtained ensure that the presented procedure needs less work in comparison with these methods and decreases considerable volume of calculation and a powerful tool for solving large amount of other problems in biology and applied mathematics. Finally the effects of the variation of all the parameters are discussed in detail.

II. MATERIALS AND METHODS

This section dealt with the Diabetes recovery model divides the Diabetes rats $N(t)$ into the three distinct subclasses, such are (i) Diabetes individuals $D(t)$ (ii) Diabetes individuals treated with plant extract $T(t)$ and (iii) have recovered from Diabetes $R(t)$.

The total variable Diabetics population size at time is defined as follows:

$$N(t) = D(t) + T(t) + R(t).$$  (1)

We consider the Diabetics rats are recruited into the population at per capita rate parameter $A$, the natural mortality rate parameter $B$ and probability of Diabetics rates $E$. The model can be represented by the following nonlinear differential equation as

$$\frac{d^2D(t)}{dt^2} = A - (B + E)D(t) + D'(t)T(t).$$  (2)

Diabetes individuals treated with plant extract $T$. The population of Diabetics rats individuals are generated at the rate parameter $B$. This population is decreased by death due to natural mortality parameter $G$, recovery rats $H$ and which disability cannot be cured parameter $E$ as mathematical model.

$$\frac{d^2T(t)}{dt^2} = B D(t) - (G + H + E)T(t) + T'(t)D(t).$$  (3)

The recovered population $R(t)$ is increased by Diabetic rates treated who recovered at the rate $G$ and decreases by the natural mortality at the rate $E$ as modeled by ordinary differential equation as,

$$\frac{d^2R(t)}{dt^2} = GT(t) − ER'(t).$$  (4)
With the initial conditions as

\[
D(0) \geq 0, T(0) \geq 0 \text{ and } R(0) \geq 0,
\]

(5)

From Eqn. (1), the total population density defined [15] as follows

\[
D(t) + T(t) + R(t) = 1 \\
\Rightarrow R(t) = 1 - D(t) - T(t).
\]

(6)

Without loss of generality we solve (2)-(5), it is enough to consider (2) and (3) as.

\[
D'(t) - D(t) T(t) + (B + E)D(t) - A = 0,
\]

(7)

\[
T'(t) - T(t) D(t) + (G + H + E)T(t) - B D(t) = 0.
\]

(8)

Where the prime (′) denotes the derivative with respective to time t.

The feasible region for the (2) and (3) defined as

\[
\Omega = \{(D(t), T(t)) \in \delta^2 \text{, } D(t) + T(t) \leq 1\}.
\]

(9)

Since,

\[
D(t) = 0 \Rightarrow \frac{dD(t)}{dt} = A > 0,
\]

and

\[
T(t) = 0 \Rightarrow \frac{dT(t)}{dt} = 0.
\]

(10)

Therefore, \(\Omega\) is positively invariant.

**Solution of Model by \(\left(\frac{\mathcal{G}}{\delta}\right)\)-expansion method**

In order to obtain an exact solution of the Eqs. (7-8) with initial conditions (5), by \(\left(\frac{\mathcal{G}}{\delta}\right)\)-expansion method [26-28].

Suppose that the solution of (7-8) can be expressed by a polynomial in \(\left(\frac{\mathcal{G}}{\delta}\right)\) as follows

\[
D(t) = a_m \left(\frac{\mathcal{G}}{\delta}\right)^m + \cdots, \text{ where } m=0,1,2,3,\ldots.
\]

(11)

\[
T(t) = b_n \left(\frac{\mathcal{G}}{\delta}\right)^n + \cdots, \text{ where } n=0,1,2,3,\ldots.
\]

(12)

Here \(a_m \neq b_n \neq 0\) and the unwritten part in (11-12) is also a polynomial in \(\left(\frac{\mathcal{G}}{\delta}\right)\), but the degree of which is generally equal to or less than \(m - 1\) and \(n - 1\), the positive integers \(m\) and \(n\) can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in (7-8).

Clearly, \(G = G(t)\) satisfies the second order linear ODE in the form,

\[
G''(t) + \lambda G'(t) + \mu G(t) = 0,
\]

(13)

where \(\lambda, \mu\) are real constants and \(G' = \frac{dG(t)}{dt}, G'' = \frac{d^2G(t)}{dt^2}\).

Further (13) can be changed into

\[
\frac{d}{dt} \left(\frac{\mathcal{G}}{\delta}\right) = - \left(\frac{\mathcal{G}}{\delta}\right)^2 - \lambda \left(\frac{\mathcal{G}}{\delta}\right) - \mu.
\]

(14)

By using (11-12) it is easily derived that,
On solving the above algebraic equations (27-30) by Maple and obtain for \(a_0, a_1, b_0\) and \(b_1\) as follows:

\[a_0 = 1,\]
\[a_1 = \frac{(B+E)-A}{\mu_1}\]

\[b_0 = -\lambda,\]

\[b_1 = -2.\]

Substituting above values in (19-20), we get

\[D(t) = 1 + \frac{(B+E)-A}{\mu_1} \left(\frac{6}{\alpha}\right) + \cdots,\] (31)

\[T(t) = -\lambda - 2 \left(\frac{6}{\alpha}\right) + \cdots,\] (32)

Substituting the general solutions of (13) into (31-32) we obtain three types of solutions of the governing equations (7-8) as follows:

(i) When \(\lambda^2 - 4\mu > 0\)

\[D_{1,1}(t) = a_0 + a_1 \left[-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2}\right] \left\{ K_1 \cosh \left(\frac{1}{2} \sqrt{\frac{\lambda^2 - 4\mu}{2}} t\right) + K_2 \sinh \left(\frac{1}{2} \sqrt{\frac{\lambda^2 - 4\mu}{2}} t\right) \right\} + \left\{ K_1 \sinh \left(\frac{1}{2} \sqrt{\frac{\lambda^2 - 4\mu}{2}} t\right) + K_2 \cosh \left(\frac{1}{2} \sqrt{\frac{\lambda^2 - 4\mu}{2}} t\right) \right\}\]

\[= 1 + \frac{(B+E)-A}{\mu_1} \left[-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2}\right] \left\{ K_1 \cosh \left(\frac{1}{2} \sqrt{\frac{\lambda^2 - 4\mu}{2}} t\right) + K_2 \sinh \left(\frac{1}{2} \sqrt{\frac{\lambda^2 - 4\mu}{2}} t\right) \right\} + \left\{ K_1 \sinh \left(\frac{1}{2} \sqrt{\frac{\lambda^2 - 4\mu}{2}} t\right) + K_2 \cosh \left(\frac{1}{2} \sqrt{\frac{\lambda^2 - 4\mu}{2}} t\right) \right\}\]

\[= 1 + \frac{(B+E)-A}{\mu_1} \left[-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2}\right] \left\{ K_1 \cosh \left(\frac{1}{2} \sqrt{\frac{\lambda^2 - 4\mu}{2}} t\right) + K_2 \sinh \left(\frac{1}{2} \sqrt{\frac{\lambda^2 - 4\mu}{2}} t\right) \right\} + \left\{ K_1 \sinh \left(\frac{1}{2} \sqrt{\frac{\lambda^2 - 4\mu}{2}} t\right) + K_2 \cosh \left(\frac{1}{2} \sqrt{\frac{\lambda^2 - 4\mu}{2}} t\right) \right\}\]

\[D_{2,1}(t) = a_0 + a_1 \left[-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2}\right] \left\{ K_1 \cos \left(\frac{1}{2} \sqrt{\frac{4\mu - \lambda^2}{2}} t\right) + K_2 \sin \left(\frac{1}{2} \sqrt{\frac{4\mu - \lambda^2}{2}} t\right) \right\} + \left\{ K_1 \sin \left(\frac{1}{2} \sqrt{\frac{4\mu - \lambda^2}{2}} t\right) + K_2 \cos \left(\frac{1}{2} \sqrt{\frac{4\mu - \lambda^2}{2}} t\right) \right\}\]

\[= 1 + \frac{(B+E)-A}{\mu_1} \left[-\frac{\lambda}{2} + \frac{\sqrt{4\mu - \lambda^2}}{2}\right] \left\{ K_1 \cos \left(\frac{1}{2} \sqrt{\frac{4\mu - \lambda^2}{2}} t\right) + K_2 \sin \left(\frac{1}{2} \sqrt{\frac{4\mu - \lambda^2}{2}} t\right) \right\} + \left\{ K_1 \sin \left(\frac{1}{2} \sqrt{\frac{4\mu - \lambda^2}{2}} t\right) + K_2 \cos \left(\frac{1}{2} \sqrt{\frac{4\mu - \lambda^2}{2}} t\right) \right\}\]

Where \(K_1\) and \(K_2\) are arbitrary constants.

(ii) When \(\lambda^2 - 4\mu < 0\)
\( T_{1,2}(t) = b_0 + b_1 \left[ -\frac{\lambda}{2} + \sqrt{\frac{\lambda^2 - 4\mu}{2}} \left\{ K_1 \cos \left( \frac{1}{2} \sqrt{\lambda^2 - 4\mu} t \right) - K_2 \sin \left( \frac{1}{2} \sqrt{\lambda^2 - 4\mu} t \right) \right\} \right] \)

\[
= -\lambda - 2 \left[ -\frac{\lambda}{2} + \sqrt{\frac{\lambda^2 - 4\mu}{2}} \left( \frac{K_1 \cos \left( \frac{1}{2} \sqrt{\lambda^2 - 4\mu} t \right) - K_2 \sin \left( \frac{1}{2} \sqrt{\lambda^2 - 4\mu} t \right)}{K_1 + tK_2} \right) \right]
\]

(iii) When \( \lambda^2 - 4\mu = 0 \)

\[ D_{1,3}(t) = a_0 + a_1 \left[ -\frac{\lambda}{2} + \frac{K_2}{K_1 + tK_2} \right] \]

\[= 1 + \left( \frac{B(E) - A}{\mu \lambda} \right) \left[ -\frac{\lambda}{2} + \frac{K_2}{K_1 + tK_2} \right] \]

\[ T_{1,3}(t) = b_0 + b_1 \left[ -\frac{\lambda}{2} + \frac{K_2}{K_1 + tK_2} \right] \]

\[= -\lambda - 2 \left[ -\frac{\lambda}{2} + \frac{K_2}{K_1 + tK_2} \right]. \]

Therefore, which are the exact solutions of the governing equations (7-8).

Notice that all solutions \((33-38)\) have been checked with Mathematica by putting them back into the governing equations \((7-8)\). It shows that the \( \left( \frac{G}{N} \right) \)-expansion method is more powerful in constructing exact solutions of \((7-8)\).

**Solution of Model by HAM**

In order to obtain an analytical solution of the non-linear differential equations \((7)\) to \((8)\) satisfying the initial conditions \((5)\), we employ Homotopy Analysis Method (HAM) \([29-30]\). This method is a recently developed approximate analytical method which is very popular amongst the present researchers. It is a special method in the sense that the pure analytic approach to solution procedure but solvable only using Computer Algebra System such as Mathematica. This method was introduced by Chinese mathematician in the literature only in 1992 by Shi Jun Liao and is described in detail \([16]\). Hence, the details are not presented here for the sake of brevity. However, the main components involved in applying HAM procedure are \((i)\) selecting suitable initial profiles and satisfying the initial conditions of the problem and \((ii)\) selecting an appropriate auxiliary linear operator so that its solutions are simpler to evaluate analytically.

In the present problem, depending upon the Eq. \((5)\), we choose the initial guesses and auxiliary linear operators as follows

\[ D_0(t) = 1 - e^t, \quad T_0(t) = 1 - e^{-t}, \]

\[ L_D = D' - D; \quad L_T = T' - T, \]

\[ L_D[C_1e^t + C_2] = 0 \quad \text{and} \quad L_T[C_3e^t + C_4] = 0. \]

In which \( C_1 \cdot s \ (i=1,2,3,4) \) are arbitrary constants. It is to be mentioned here that the choice of initial profiles and linear operators are not unique for a given problem, but the faster convergence of the solution depends on the choice of those.

**Zeroth and Higher-order deformation Equations**

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[483]
To obtain the HAM solution for the governing Eqns. (7) - (8), in the standard notations followed in any HAM Analysis, let \( \gamma \in [0, 1] \) be an embedding parameter and \( c_D \) and \( c_T \) are the convergence control parameters(CCP). Then the zeroth-order deformation equation takes the form [30] as,

\[
(1 - \gamma) L_0 [\tilde{D}(t, \gamma) - D_0(t)] = \gamma c_D \mathcal{N}_0 [\tilde{D}(t, \gamma), \tilde{T}(t, \gamma)],
\]

\[
(1 - \gamma) L_T [\tilde{T}(t, \gamma) - T_0(t)] = \gamma c_T \mathcal{N}_T [\tilde{T}(t, \gamma), \tilde{D}(t, \gamma)],
\]

where

\[
\mathcal{N}_0 [\tilde{D}(t, \gamma), \tilde{T}(t, \gamma)] = \frac{\partial^2 \tilde{D}(t, \gamma)}{\partial \gamma^2} + \left( \frac{\partial \tilde{D}(t, \gamma)}{\partial \gamma} \right) \tilde{T}(t, \gamma) + \left( B(\gamma) + E(\gamma) \right) \tilde{D}(t, \gamma) - A(\gamma),
\]

\[
\mathcal{N}_T [\tilde{T}(t, \gamma), \tilde{D}(t, \gamma)] = \frac{\partial^2 \tilde{T}(t, \gamma)}{\partial \gamma^2} + \left( \frac{\partial \tilde{T}(t, \gamma)}{\partial \gamma} \right) \tilde{D}(t, \gamma) + \left( G(\gamma) + H(\gamma) + E(\gamma) \right) \tilde{T}(t, \gamma) - B(\gamma) \tilde{D}(t, \gamma).
\]

For \( l \)th-order deformations equation, we first differentiate Eqs. (43)-(44) \( l \)-times with respect to \( \gamma \); dividing them by \( l! \) and then set \( \gamma = 0 \). Following this procedure we have

\[
L_0 [D_l(t) - \Omega_l D_{l-1}(t)] = c_D \mathcal{R}_D \left( \tilde{T}(\eta, \gamma), \tilde{D}(\eta, \gamma) \right),
\]

\[
L_T [T_l(t) - \Omega_l T_{l-1}(t)] = c_T \mathcal{R}_T \left( \tilde{T}(\eta, \gamma), \tilde{D}(\eta, \gamma) \right).
\]

Where \( \mathcal{R}_D(\tilde{T}(\eta, \gamma), \tilde{D}(\eta, \gamma)) \) and \( \mathcal{R}_T(\tilde{T}(\eta, \gamma), \tilde{D}(\eta, \gamma)) \) are remainders term of the linear operators such as,

\[
\mathcal{R}_D \left( \tilde{T}(\eta, \gamma), \tilde{D}(\eta, \gamma) \right) = D^{l-1}_D(t) + \sum_{j=0}^{l-1} [D_j(t) T_{l-1-j}(t)] + (B(\gamma) + E(\gamma)) D_{l-1}(t)
\]

\[
\mathcal{R}_T \left( \tilde{T}(\eta, \gamma), \tilde{D}(\eta, \gamma) \right) = T^{l-1}_D(t) + \sum_{j=0}^{l-1} [T_j(t) D_{l-1-j}(t)] - B(\gamma) D_{l-1}(t) + (G(\gamma) + H(\gamma) + E(\gamma)) T_{l-1}(t).
\]

Where \( \Omega_l \) is defined as \( \Omega_l = \{ 0, 1 \leq 1, 1 > l \} \).

Expanding \( \tilde{D}(t, \gamma) \) and \( \tilde{T}(t, \gamma) \) in Taylor series with respect to \( \gamma \),

\[
\tilde{D}(t, \gamma) = D_0(t) + \sum_{i=1}^{\infty} D_i(t) \gamma^i,
\]

\[
\tilde{T}(t, \gamma) = T_0(t) + \sum_{i=1}^{\infty} T_i(t) \gamma^i,
\]

where, \( D_i(t) = \frac{1}{\gamma^i} \left. \frac{\partial^i \tilde{D}(t, \gamma)}{\partial \gamma^i} \right|_{\gamma=0} \).

For \( \gamma = 0 \) and \( \gamma = 1 \) in Eqs. (47)-(48) one may be write as,

\[
\tilde{D}(t, 0) = D_0(\eta), \quad \tilde{D}(t, 1) = D(t),
\]

\[
\tilde{T}(t, 0) = T_0(\eta), \quad \tilde{T}(t, 1) = T(t).
\]

Thus as \( \gamma \) increases from 0 to 1 and \( \tilde{D}(t, \gamma) \) and \( \tilde{T}(t, \gamma) \) varies from the initial guess, the functions \( D_0(t) \) and \( T_0(t) \) approaches to the solution \( D(t) \) and \( T(t) \) of the governing equations respectively. Here, the auxiliary parameters are suitably chosen so that the series solution converges for \( \gamma = 1 \).

\[
D(t) = D_0(t) + \sum_{i=1}^{\infty} D_i(t),
\]
\begin{equation}
T(t) = T_0(t) + \sum_{l=1}^{\infty} T_l(t).
\end{equation}

Therefore we get the general approximate analytical solutions \(D_l\) and \(T_l\) in terms of series solutions evaluated up to \(l^{th}\)-order \(D_l^*\) and \(T_l^*\) along with the solution of the linear operator chosen in the problem as

\begin{align*}
D_l(t) &= D_l^*(t) + C_1 e^t + C_2, \\
T_l(t) &= T_l^*(t) + C_3 e^t + C_4.
\end{align*}

We solve the Eqns. (58) - (59) for various values of \(l\) starting from 1, 2, 3, … by means of the symbolic computation software Mathematica 10.0.

**Computational Procedure**

A Mathematica code is written to solve the Eqns. (58) to (59). Starting from known functions for \(D_0(t)\) and \(T_0(t)\), the next set of functions \(D_1(t)\) and \(T_1(t)\) are computed using the (58) to (59). Using these two sets, the next set of functions \(D_2(t)\) and \(T_2(t)\) are algebraically computed and so on. At any stage, the functions obtained are a polynomial of degree \(l\) in ‘\(c_D\)’ and ‘\(c_T\)’ are the convergent control parameters, where the coefficients of the polynomial may contain some or all the parameters present in the problem. The computations are carried out up to \(l = 50\), to as certain that the values of \(D_l(t)\) and \(T_l(t)\) obtained at that stage are consistent and accurate up to 7 digits. It is to be mentioned here that the analytical expressions obtained for \(D_l^*(t)\) and \(T_l^*(t)\) are too lengthy even for the value of \(l = 5\). Hence the actual expressions are reproduced here.

**Convergence of the solutions**

As it is done traditionally in homotopy analysis, the auxiliary parameters ‘\(c_D\)’ and ‘\(c_T\)’ are optimized by drawing these curves with respect to \(c_D\) and \(c_T\) to find the convergent interval for \(D(t)\) and \(T(t)\). From these curves it is found that the admissible ranges of \(c_D\) and \(c_T\) are \(-2.3 < c_D < -0.3\) and \(-1.5 < c_T < -0.5\). The \(c_D\) and \(c_T\)-curves are shown in Figure 1. It is clear from Figure 1 that the numerical values for \(D(t)\) and \(T(t)\) are insensitive to the value of \(c_D\) and \(c_T\) chosen from the above mentioned range.

The results are presented in the form of tables and figures in the following sections. In all of the figures, we use \(B = 150\), \(E = 13\), \(G = A = 1\), \(-c_D = 0.741625\) and \(-c_T = 0.627879\) unless it is mentioned otherwise.

**Table 1. The laboratory experimental values of D(t) at t=90.**

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Laboratory experimental values of D(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>196.3826822</td>
</tr>
<tr>
<td>Control</td>
<td>411.1954257</td>
</tr>
<tr>
<td>100mg/kg</td>
<td>737.0110911</td>
</tr>
<tr>
<td>250mg/kg</td>
<td>114.3107323</td>
</tr>
<tr>
<td>Tolbutamide</td>
<td>516.2874261</td>
</tr>
</tbody>
</table>
Table 2. The \( \left( \frac{G'}{G} \right) \)-expansion method* and HAM** solutions for \( D(t) \) at \( t=90 \) when \( \mathbf{A} = 1, \mathbf{E} = 13, \) \(-c_D = 0.600425 \) and \(-c_T = 0.627879.\)

<table>
<thead>
<tr>
<th>The population of Diabetics rats individuals are generated at the rate parameter B</th>
<th>*Results of ( D(90) )</th>
<th>**Approximate analytical values of ( D(90) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>196.07</td>
<td>196.05756209</td>
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<tr>
<td>140</td>
<td>251.80</td>
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<td>160</td>
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<td>180</td>
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<tr>
<td>220</td>
<td>517.14</td>
<td>516.30751646</td>
</tr>
</tbody>
</table>

Table 3. The laboratory experimental values of \( T(t) \) at \( t=90 \).

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Laboratory experimental values of ( T(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>124.188922</td>
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<tr>
<td>250mg/kg</td>
<td>329.935951</td>
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<tr>
<td>Tolbutamide</td>
<td>432.806407</td>
</tr>
</tbody>
</table>

Table 4. The \( \left( \frac{G'}{G} \right) \)-expansion method* and HAM** solutions for \( T(t) \) at \( t=90 \) when \( \mathbf{B} = 150, \mathbf{E} = 13, \mathbf{G} = \mathbf{A} = 1, \) \(-c_D = 0.600425 \) and \(-c_T = 0.627879.\)

<table>
<thead>
<tr>
<th>Recovery rats parameter H</th>
<th>*Values of ( T(90) )</th>
<th>*Approximate analytical values of ( T(90) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>122.06</td>
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<td>330.10820800</td>
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<tr>
<td>30</td>
<td>432.93</td>
<td>432.80355279</td>
</tr>
</tbody>
</table>
Fig. 1 $c_D$ and $c_T$-curve for the functions $D'(t)$ and $T'(t)$ at $t = 0$.

Fig. 2 Residual Error analysis curves.

Fig. 3 $D(t)$-curve for the time $t$ at $c_D = -0.741625$ and $c_T = 0.627879$. 

Residual Error Analysis

In order to choose a proper value of CCP, the squared residual error is defined

\[ E_D \approx \frac{1}{\ell} \sum_{j=0}^{\ell} \left[ N \left( \sum_{i=0}^{\ell} D_i(j\Delta x) \right) \right]^2, \]  
\[ \text{here } \Delta x = 10/\ell \text{ and } \ell = 25. \]  
\[ (60) \]

\[ E_T \approx \frac{1}{\ell} \sum_{j=0}^{\ell} \left[ N \left( \sum_{i=0}^{\ell} T_i(j\Delta x) \right) \right]^2, \]  
\[ \text{here } \Delta x = 10/\ell \text{ and } \ell = 30. \]  
\[ (61) \]

From Eqs. (60) and (61) to find \( c_D \) and \( c_T \) values and obtained these values are used for the entire computations of solution. The HAM based Mathematica package BVPh 2.0 has been utilized to compute the average residual errors of governing equations. Comparison between \( c_D \) and \( c_T \) error analysis, it shows very accurate results have seen in Fig.2. The corresponding residual error decreases very rapidly. It is found to be safety, efficiency and accuracy in Diabetic.

III. RESULTS AND DISCUSSION

The analytical solutions obtained are a set of polynomials in higher powers of \( c_D \) and \( c_T \) (the Homotopy parameters) whose coefficients contain A, B, E, G, H and E are mentioned above. The solution can be obtained up to any order depending upon the required accuracy. It is often the case that the expressions obtained from HAM are too lengthy even for a polynomial of degree 5 in \( c_D \) and \( c_T \). In the present analysis, the computations have been performed up to 25\(^{th}\) degree to ascertain the consistency in values, but the results presented in figures are calculated from the 20\(^{th}\) degree polynomial in \( c_D \) and \( c_T \) of the series solution containing all the parameters.

In order to assess the accuracy of present \( \left( \frac{c}{T} \right)\)-expansion method and HAM solutions for \( D(t) \) and \( T(t) \), compared with laboratory experimental values shown in Tables 1-4. It is seen from these tables that, the calculative values of Homotopy analysis at 1% level of significance is less than the laboratory experimental values and hence there is no change in Blood Glucose and Blood Glucose Tolerance. The oral glucose to tolerance test (OGTT) measures the body’s ability to use a type of sugar, called glucose that is the body’s main source of energy. The extract (Dose 150mg/kg and Dose 300mg/kg) prevented the increase in the Blood glucose levels significantly after glucose load in three selected medicinal plants. Tables 2, 3 illustrates that, the population of Diabetics rats individuals are generated at the rate parameter B and Recovery rats parameter H are increases as values of \( D(t) \) and \( T(t) \) at \( t=90 \) increase. It is also observed that the maximum glucose tolerance is \( H. \ isora \) extract at the 90\(^{th}\) mins after the glucose loading followed by \( P. \ oleracea \) and \( C. \ attenuate \). Only one set of representative result for \( D(t) \) and \( T(t) \)-curves for a various value for the natural mortality rate parameter B are presented in Figures. 3-4. It is observed that, \( D(t) \) and \( T(t) \)-curves increases as natural mortality rate
parameter B increases due to probability of Diabetics rates E. The probability of Diabetics rates E is fixed at 13.0.

IV. CONCLUSION
The different extract of H. isora, P. oleracea and C. attenuata were tested for anti-Diabetic activity, earlier by glucose tolerance test and this hypothesis conform by \( \frac{IC_{50}}{C} \)-expansion method and Homotopy Analysis Method (HAM). However, present work will enhance the possible mechanism of action of the different extract for their pharmacological response with help of these two techniques and it confirmed the evidence which was previously scientific validate on animal anti-Diabetics. This novelty will enhance their claim to be used regular anti-Diabetics.

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VI. REFERENCES


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