In recent year’s many special functions given by mathematicians, here a new function termed as Advanced Modified M- function has been introduced. This Function is a particular case of H-function given in [2,3]. This function is important because hypergeometric function and Mittag-Leffler function follow as particular cases and these functions have great significance in the context of problems in physics, biology, engineering and applied sciences. It is to be noted that Mittag-Leffler [4,5] function occurs as solution of fractional integral equations in those subjects. In this paper we have also obtained the fractional integration and fractional differentiation of Advanced Modified M-function.

Mathematics Subject Classification: 33C60, 33E12, 82C31, 2

ABSTRACT

In this section, we define Advanced Modified M- function, which is the most generalization of New Generalized Mittag-Leffler Function. Here, we give first the notation and the definition of the New Special Advanced Modified M- function, introduced by the author as follows:

\[
a_{\beta,\gamma,\delta,\rho} M_{\rho}^{k_1,...,k_p,l_1,...,l_q,c}(t) = \sum_{n=0}^{\infty} \frac{(a_1)_n \cdots (a_p)_n (y)_n (\delta)_n k_1^n \cdots k_p^n}{(b_1)_n \cdots (b_q)_n (\rho)_n l_1^n \cdots l_q^n} \prod_{i=1}^{p} a_i^{a_i} \prod_{i=1}^{q} b_i^{b_i} \frac{1}{n!} \Gamma((n+\gamma)\alpha-\beta) t^n
\]

RELATIONSHIP OF THE \(a_{\beta,\gamma,\delta,\rho} M_{\rho}^{k_1,...,k_p,l_1,...,l_q,c}(t)\) FUNCTION AND OTHER SPECIAL FUNCTIONS

In this section, we define relationship of M function and various special functions

(1) If \(\frac{\prod_{i=1}^{p} a_i^{a_i}}{\prod_{i=1}^{q} b_i^{b_i}} = 1\) Then (1.1) Advanced Modified M- function converts into M-function, introduced by the author as follows:

\[
a_{\beta,\gamma,\delta,\rho} M_{\rho}^{k_1,...,k_p,l_1,...,l_q,c}(t) = \sum_{n=0}^{\infty} \frac{(a_1)_n \cdots (a_p)_n (y)_n (\delta)_n k_1^n \cdots k_p^n}{(b_1)_n \cdots (b_q)_n (\rho)_n l_1^n \cdots l_q^n} \frac{(ct)^{(n+\gamma)\alpha-\beta-1}}{n!} \Gamma((n+\gamma)\alpha-\beta) t^n
\]
(2). For \(k_1 = k_2 \ldots k_p = 1, l_1, \ldots l_q = 1, \delta = 1 \) and \(\rho = 1, c = 1\), we defined relationship of

**Advanced Modified M – function** and various special functions.

The **Advanced Modified M – function** reduces to New Generalized Mittag-Leffler Function [6]

\[
a_{\alpha, \gamma, 1, 1} \mathbb{M}^n_1(t) = t^{\alpha - \beta - 1} \sum_{n=0}^{\infty} \frac{(\gamma)_n (a^n(t))^n}{n! \Gamma((n + \gamma)\alpha - \beta)} = t^{\alpha - \beta - 1} E_{\alpha, \gamma - \beta}^{\alpha} [at^\alpha] \tag{2}
\]

(ii). We take \(\gamma = 1\), in (2) obtained Generalized Mittag-Leffler Function [10], we get

\[
a_{\alpha, 1, 1, 1} \mathbb{M}^n_1(t) = \sum_{n=0}^{\infty} \frac{(a^n(t))^{n\alpha}}{\Gamma(n\alpha + 1)} = E_{\alpha} [at^\alpha] \tag{3}
\]

(iii). Further \(\beta = \alpha - 1,\) in (3), this \(\mathbb{M}\) function converts Mittag-Leffler Function [6,7], we have

\[
a_{\alpha, \alpha - 1, 1, 1} \mathbb{M}^n_1(t) = \sum_{n=0}^{\infty} \frac{(a^n(t))^{n\alpha}}{\Gamma(n\alpha + 1)} = E_{\alpha} [at^\alpha] \tag{4}
\]

(iv). When \(\alpha = 1\) and \(\beta = \alpha - \beta\) in (4) then the \(\mathbb{M}\) function treats as Agarwal’s Function [1]

\[
a_{\alpha, \alpha - \beta, 1, 1} \mathbb{M}^n_1(t) = \sum_{n=0}^{\infty} \frac{(t)^{n\alpha + \beta - 1}}{\Gamma(n\alpha + \beta)} = E_{\alpha, \beta} [t^\alpha] \tag{5}
\]

(ix). On substituting \(\alpha = 1, \beta = -\beta\) in (3), we get Miller and Ross Function [5].

\[
a_{1, -\beta, 1, 1} \mathbb{M}^n_1(t) = \sum_{n=0}^{\infty} \frac{(a^n(t))^{n+\beta}}{\Gamma(n + \beta + 1)} = E_{\beta, \alpha} \tag{6}
\]

**MATHEMATICAL PREREQSITIES**

The Riemann-Liouville fractional integral of order \(\nu \in C\) is defined by Miller and Ross (1993, p.45)

\[
o D_t^{-\nu} f(t) = \frac{1}{\Gamma(\nu)} \int_0^t (t-u)^{\nu-1} f(u) du, \tag{3.1}
\]

where Re(\(\nu\))>0. Following Samko et al. (1993, p. 37) we define the fractional derivative for \(\alpha > 0\) in the form

\[
o D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_0^t (t-u)^{\alpha-n+1} f(u) du, \tag{3.2}
\]

\(n = [\text{Re}(\alpha)] + 1\),

Where \([\text{Re}(\alpha)]\) means the integral part of \(\text{Re}(\alpha)\).
fractional integral and fractional differential of the
Advanced Modified M – Function

Let us consider the fractional Riemann – Liouville (R-L) integral operator, as in [5,8] (for lower limit α = 0 with respect to variable z) of the **Advanced Modified M – function** (1).

\[
\begin{align*}
0D_\gamma^{a, b, \nu, \rho} M_q^{k_1, \ldots, k_n, \lambda_1, \ldots, \lambda_n}(t) &= \frac{1}{\Gamma(v)} \sum_{n=0}^{\infty} \frac{(a_1)_n \ldots (a_p)_n (y)_n (\delta)_n \Pi_{i=1}^{n} a_{q_i}}{(b_1)_n \ldots (b_q)_n (\rho)_n \Pi_{i=1}^{n} b_{l_i}} k_1^n \ldots k_p^n (c)^{(n+\nu)\alpha-\beta-1} \times \\
& \quad \int_0^{z} (z-t)^{\nu-1} (t)^{(n+\nu)\alpha-\beta-1} dt \\
& = \frac{1}{\Gamma(v)} \sum_{n=0}^{\infty} \frac{(a_1)_n \ldots (a_p)_n (y)_n (\delta)_n \Pi_{i=1}^{n} a_{q_i}}{(b_1)_n \ldots (b_q)_n (\rho)_n \Pi_{i=1}^{n} b_{l_i}} k_1^n \ldots k_p^n (c)^{(n+\nu)\alpha-\beta-1} \times \\
& \quad \frac{B((n+\nu)\alpha-\beta, \nu)}{\Gamma((n+\nu)\alpha-\beta)} \\
& = \frac{\Gamma((n+\nu)\alpha-\beta-\nu)}{\Gamma((n+\nu)\alpha-\beta+v-n)} \times \frac{((n+\nu)\alpha-\beta-n)_n}{((n+\nu)\alpha-\beta+v-n)_n}
\end{align*}
\]

http://www.ijesrt.com
\[ 0D_{t}^{-\alpha\beta,\gamma,\delta,\rho}_{pM_{q}^{k_{1},\ldots,k_{p},l_{1},\ldots,l_{q},c}}(t) = \frac{\Gamma((n + \gamma)a - \beta - n)}{\Gamma((n + \gamma)a - \beta + \nu - n)} z^{\nu} \frac{\alpha,\beta,\gamma,\delta,\rho}{}_{pM_{q}^{k_{1},\ldots,k_{p},l_{1},\ldots,l_{q},c}} (a_{1}, \ldots, a_{p}, \gamma, \delta, ((n + \gamma)a - \beta - n); b_{1}, \ldots, b_{q}, \rho, ((n + \gamma)a - \beta + \nu - n); z) \] (4.1)

Riemann – Liouville Fractional derivative of Advanced Modified M – function which indices \( p, q \) are increased to \( (p + 1), (q + 1) \).

Analogously, Riemann – Liouville fractional derivative operator \([5,8]\) of the \( M - \text{Function with respect to } z \).

\[ D_{z}^{\alpha,\beta,\gamma,\delta,\rho}_{pM_{q}^{k_{1},\ldots,k_{p},l_{1},\ldots,l_{q},c}}(t) = \frac{1}{\Gamma(n - v)} \left( \frac{d}{dz} \right)^{n} \int_{0}^{z} (z - t)^{n-v-1} \frac{a_{1}^{\alpha} \ldots a_{p}^{\alpha} (\gamma)^{n} \prod_{i=1}^{n} a_{i}^{a_{i}} \prod_{i=1}^{n} b_{i}^{b_{i}} \prod_{i=1}^{m} a_{i}^{a_{i}} \prod_{i=1}^{m} b_{i}^{b_{i}} \prod_{i=1}^{n} (\delta)^{n} \prod_{i=1}^{m} (\rho)^{m} \prod_{i=1}^{n} l_{i}^{l_{i}} \prod_{i=1}^{m} l_{i}^{l_{i}} n! \Gamma((n + \gamma)a - \beta)}{n! \Gamma((n + \gamma)a - \beta)} \right) dt \]

\[ = \frac{1}{\Gamma(n - v)} \sum_{n=0}^{\infty} \frac{(a_{1})_{n} \ldots (a_{p})_{n} (\gamma)_{n} \prod_{i=1}^{n} a_{i}^{a_{i}} \prod_{i=1}^{n} b_{i}^{b_{i}} \prod_{i=1}^{m} a_{i}^{a_{i}} \prod_{i=1}^{m} b_{i}^{b_{i}} \prod_{i=1}^{n} (\delta)_{n} \prod_{i=1}^{m} (\rho)^{m} \prod_{i=1}^{n} l_{i}^{l_{i}} \prod_{i=1}^{m} l_{i}^{l_{i}} n! \Gamma((n + \gamma)a - \beta)}{n! \Gamma((n + \gamma)a - \beta)} \int_{0}^{z} (z - t)^{n-v-1} \right( \frac{z}{(n + \gamma)a - \beta} \right) \left( \frac{d}{dz} \right)^{n} \]

We use the modified Beta-function:

\[ \int_{a}^{b} (b - t)^{-1} (t - a)^{\alpha-1} dt = (b - a)^{\alpha+\beta-1} B(\alpha, \beta), \text{ for } R(\alpha) > 0, R(\beta) > 0 \]
\[
\frac{1}{\Gamma(n - \nu)} \sum_{n=0}^{\infty} \frac{(a_1)_n \cdots (a_p)_n (\gamma)_n (\delta)_n \prod_{i=1}^{n} a_i^{\alpha_i} k_i^{\nu} \cdots k_p^{\nu} }{(b_1)_n \cdots (b_q)_n (\rho)_n \prod_{i=1}^{n} b_i^{\beta_i} l_i^{\rho_1} \cdots l_q^{\rho_q} } n! \Gamma((n + \gamma)\alpha - \beta)
\]

\[
\times \left( cz^{(n+\gamma)\alpha - \beta - v} \right)^{(n+\gamma)\alpha - \beta - v - 1}
\]

Where \( k + 1 > 0, n - \nu > 0 \)

Differentiation \( n \) times the term \( z^{(n+\gamma)\alpha - \beta + n - v - 1} \) and using again

\[
\Gamma(\alpha + k) = (a)_k \Gamma(\alpha), \text{ representation (4.2) reduces to}
\]

\[
= \frac{1}{\Gamma(\alpha(n + \gamma) - \beta + n)} \sum_{n=0}^{\infty} \frac{(a_1)_n \cdots (a_p)_n (\gamma)_n \prod_{i=1}^{n} a_i^{\alpha_i} (\delta)_n k_i^{\nu} \cdots k_p^{\nu} 1 }{(b_1)_n \cdots (b_q)_n (\rho)_n \prod_{i=1}^{n} b_i^{\beta_i} l_i^{\rho_1} \cdots l_q^{\rho_q} } n!
\]

\[
\times \left( c z^{(n+\gamma)\alpha - \beta - v} \right)^{(n+\gamma)\alpha - \beta - v - 1}(n + \gamma)\alpha - \beta - v \right)
\]

\[
\frac{1}{\Gamma(\alpha(n + \gamma) - \beta + n)} \left( cz^{(n+\gamma)\alpha - \beta - v} \right)^{(n+\gamma)\alpha - \beta - v - 1}
\]

\[
\left( cz^{(n+\gamma)\alpha - \beta - v} \right)^{(n+\gamma)\alpha - \beta - v - 1}
\]

(4.3) gives a Riemann – Liouville fractional derivative of Advanced Modified M – function, which indices \( p, q \) are increased to \( (p+1),(q+1) \).

**CONCLUSION**

In this present work, we have introduced a new special function and two new results in which we have obtained fractional Integration and Fractional Differentiation of Advanced Modified M – Function.

**REFERENCES**


