Abstract
Pressure regulating valve is a mechanical device consist of spool and spring, manage the valve outlet pressure is the main function of pressure regulating valve. In this paper dynamic model is created, SIMULINK-Matlab is used to simulate the valve; the results show the spool position for linear spring and the nonlinear spring/damper cases valve displacement behavior.

Keywords: Pressure regulator, dynamic, mechanical model, response, non-linear spring, non-linear damper.

Introduction
Pressure regulating valve or pressure reducing is a mechanical device used to fix fluid pressure, the valve consists of moving plunger attached to spring, the valve internal components are modelled as spring mass damper system.

Pressure regulator is studied based on the variation of fluid temperature and the variations in its physical properties [1]. The modelling of control valves is created based on valves operations and non-linearity of the valves internal components [2], the work is divided to three parts, the first part for studying the control valve basic components, the second part for control valve mathematical modelling, and the third part for simulation results and analysis. [3] The MATLAB/SIMULINK software is used to simulate pressure relief valve, the study showed the effect of oil compressibility on the valve operation, the software results are the relief valve piston displacement and the valve outlet flow rate. [4] A normal valve is modelled in his study, the valve is used to control the operation of drill, and the study showed the mathematical model of the valve internal parameters and the valve exit pressure. A mathematical dynamic model of a hydraulic pressure relief valve [5] is created; the mathematical model is created to show the effect of fluid incompressibility and valve internal parameters on the valve operation. The transient response of pressure relief valve is shown [3], [7]: the results are produced using MATLAB software, graphical results showed the transient and full response of the valve internal mass-spring-damper unit. [6] A mathematical model is deduced for the hydraulic braking valve, the simulation program is developed using the MATLAB-SIMULINK where the mathematical model is implemented in the program. The program is used to study the dynamic valve response due to different input displacement values and effect of varying different design parameters on the overall performance of valve. The results showed the valve response for both linear and nonlinear elements.

Valve Modeling
Pressure regulating valve has two ports, one for high pressure side and the second one for low pressure side, figure (1) shows real pressure regulating valve.

Figure 1. Pressure regulating valve ports.
The high pressure side is connected to the pump and the low pressure side is connected to tank, as pressure increased in the circuit above the desired value the valve piston moves upward so the low pressure side is connected to high pressure side and the pressure inside the circuit remains constant, the pump flow rate is $Q_p$, load flow rate is $Q_L$, $Q_C$ is the control flow rate, $Q_v$ is the valve flow rate, $P_1$ is the supply pressure, and $P_2$ is the load pressure, figure (2) shows the flow the pressure regulating valve circuit.

**Figure 2. Pressure regulating valve circuit.**

The symbols of the mathematical model of the pressure regulating valve are listed in table (1).

**Table 1. Pressure regulating Valve model symbols.**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$d_v$</td>
<td>Valve diameter (m).</td>
</tr>
<tr>
<td>$d_o$</td>
<td>Orifice diameter (m).</td>
</tr>
<tr>
<td>$A_v$</td>
<td>Valve flow area (m²).</td>
</tr>
<tr>
<td>$\beta_e$</td>
<td>Oil Bulk modulus (Pa).</td>
</tr>
<tr>
<td>$F$</td>
<td>Spring preload (N).</td>
</tr>
<tr>
<td>$C_d$</td>
<td>Orifice discharge coefficient.</td>
</tr>
<tr>
<td>$k$</td>
<td>Spring stiffness (N/m).</td>
</tr>
<tr>
<td>$C$</td>
<td>Valve damping coefficient (N.s/m).</td>
</tr>
<tr>
<td>$m$</td>
<td>Valve spool mass (kg).</td>
</tr>
<tr>
<td>$P_o$</td>
<td>Atmospheric pressure (Pa).</td>
</tr>
<tr>
<td>$P_1$</td>
<td>Supply pressure (Pa).</td>
</tr>
<tr>
<td>$P_2$</td>
<td>Load pressure (Pa).</td>
</tr>
<tr>
<td>$Q_c$</td>
<td>Control flow (m³/sec).</td>
</tr>
<tr>
<td>$Q_L$</td>
<td>Load flow (m³/sec).</td>
</tr>
<tr>
<td>$Q_v$</td>
<td>Valve flow (m³/sec).</td>
</tr>
<tr>
<td>$V_1$</td>
<td>Valve inlet volume (m³).</td>
</tr>
<tr>
<td>$V_2$</td>
<td>Valve load volume (m³).</td>
</tr>
<tr>
<td>$x$</td>
<td>Valve motion (m).</td>
</tr>
<tr>
<td>$\dot{x}$</td>
<td>Valve velocity (m/sec).</td>
</tr>
<tr>
<td>$\ddot{x}$</td>
<td>Valve acceleration (m/sec²).</td>
</tr>
<tr>
<td>$\dddot{x}$</td>
<td>Inlet pressure rise rate (Pa/sec).</td>
</tr>
<tr>
<td>$\dddot{p}_1$</td>
<td>Load pressure rise rate (Pa/sec).</td>
</tr>
</tbody>
</table>

The free body diagram of the plunger is shown in figure (3), the figure shows the forces on the valve plunger in x direction, the supply pressure force in the direction of motion, spring force, damper force, and load pressure force are on the opposite direction of motion.

**Figure 3. Pressure regulating valve plunger free body diagram.**

The newton second law shows the summation of forces in the direction of motion:

$$\sum F_x = m \dddot{x}$$  \hspace{1cm} (1)

$$P_1 A - F_x - k x^\alpha - c \dot{x}^\beta = m \dddot{x}$$  \hspace{1cm} (2)

Where $\alpha$ and $\beta$ are the nonlinear spring and damper elements exponents.
From equation (2) the supply pressure can be written as:

\[ P_1 = \frac{F_x + kx^\alpha}{A} \]  

(3)

The control valve flow rate \( Q_c \) is found from equation (4) based on flow area, plunger velocity, fluid bulk modulus, and supply pressure variations:

\[ Q_c = A\dot{x} + \frac{V_1 dP_1}{\beta \, dt} \]  

(4)

The variation in supply pressure with respect to time is found from equation (5):

\[ \frac{dP_1}{dt} = \frac{\beta}{V_1} (Q_c - A\dot{x}) \]  

(5)

The control valve flow rate is found also from equation (6) based on the orifice discharge coefficient \( C_d \), supply pressure \( P_1 \), and load pressure \( P_2 \):

\[ Q_c = C_d A_c \sqrt{\frac{2}{\rho} \sqrt{P_2 - P_1}} \]  

(6)

The valve flow rate \( Q_v \) depends on load pressure \( P_2 \) and valve flow cross sectional area \( A_v \) as shown in equation (7):

\[ Q_v = C_d A_v \sqrt{\frac{2}{\rho} \sqrt{P_2 - P_1}} \]  

(7)

The total flow rate or the pump flow rate is the summation of control, load, and valve flow rates pulse the effect of load pressure variations:

\[ Q_p = Q_c + Q_l + Q_v + \frac{V_2 dP_2}{\beta \, dt} \]  

(8)

**Simulation using Matlab**

The pressure regulating value is simulated using SIMULINK software, figure (4) shows the system diagram in SIMULINK software, and also the physical parameters are listed in table (2).

Valve position or displacement verses time (x) is the major result from the simulation, valve displacement for linear damping and stiffness elements is shown in figure (5), in figure (6) show the effect of the nonlinear stiffness when \( \alpha = 3.0 \), for \( \alpha = 0.3 \) the valve displacement with time is plotted in figure (7). The effects of non-linear damper is presented in figures (8) and (9), figure (8) presents the valve displacement with time for \( \beta = 3.0 \) also for \( \beta = 0.3 \) the valve response is shown in figure (9).

The results show that the stiffness effects on the valve response when \( \alpha < 1 \), the vibration amplitude is dropped, another side the there is no effect of the damper non-linearity on the valve response.

**Table 2. Pressure regulating valve physical parameters.**
Valve diameter \( d_v \) | 0.018 m. \\
Orifice diameter \( d_o \) | 0.001 m. \\
Oil Bulk modulus \( \beta_e \) | \( 1 \times 10^7 \) Pa. \\
Spring preload \( F \) | 5000 N. \\
Orifice discharge coefficient \( C_d \) | 0.6 \\
Spring stiffness \( k \) | 210000 N/m. \\
Valve damping coefficient \( C \) | N.s/m. \\
Valve spool mass \( m \) | 0.05 kg. \\
Atmospheric pressure \( P_o \) | 101300 Pa. \\
Valve inlet volume \( V_1 \) | 0.000005 m\(^3\). \\
Valve load volume \( V_2 \) | 0.0001 m\(^3\).

Conclusions

In this paper a non-linear dynamic model of pressure regulating valve is presented, the vibration model shows the internal valve components, the valve is modelled as mass, spring, and damper system, the supply pressure and load pressure are the external forces effect on the system, the variations in load and supply pressures reflects on the plunger displacement.

The study shows that valve displacement with time for certain load and supply conditions, also load pressure and inlet flow rate are plotted with time for certain running conditions. The mathematical model and SIMULINK analysis shows valve displacement with time for linear and non-linear elements, the non-linear spring with \( \alpha<1 \) shows good effect on the valve response.

References

Figure 5. Linear Valve displacement vs. time.
Figure 6. Valve displacement vs. time for non-linear spring with $\alpha=3.0$ and $\beta=1$. 
Figure 7. Valve displacement vs. time for non-linear spring with $\alpha=0.3$ and $\beta=1$.
Figure 8. Valve displacement vs. time for non-linear damper with $\alpha=1.0$ and $\beta=3.0$
Figure 9. Valve displacement vs. time for non-linear damper with $\alpha=1.0$ and $\beta=0.3$. 