A theoretical model presented here is to study the effect of thermal gradient on vibration of visco-elastic rhombic plate of variable thickness. Temperature and thickness vary linearly along the x-direction. Rayleigh-Ritz technique is used to obtain a frequency equation with a two-term deflection function. Assumed that the plate is clamped on all the four edges. Numeric values of frequencies are calculated for different values of thermal gradient, taper parameter and skew angle for both the modes of vibration. The results are presented in tabular forms.

Plates are inherently associated with many mechanical structures which are designed to perform under extreme dynamic loading conditions. There has been a constant need for the lightweight and high strength materials for various applications like aerospace and automobiles. Therefore visco-elastic materials are utilized in making structural parts of equipment used in modern technological industries. Further, in mechanical systems where certain parts of machine have to operate under elevated temperature, its effect is far from negligible. The reason for this is that during heated up period of structures exposed to high intensity heat fluxes, the material properties undergo significant vibration.

The Rayleigh-Ritz technique has been used to determine the frequencies equation of the plate. Due to temperature variation, we assume that non-homogeneity occurs in Modulus of Elasticity. The frequency to the first two modes of vibration is obtained for a clamped Rhombic plate for various values of thermal gradient (α), taper constant (β) and skew angle (θ). All the numeric calculation has been done for Duralium, an alloy of Aluminium.

\[ x' = x - y \tan \theta \quad \text{and} \quad y' = y \sec \theta \quad (1) \]

The boundaries of the plate in oblique co-ordinate are

\[ x' = 0, x' = a \quad \text{and} \quad y' = 0, y' = b \quad (2) \]

It is assumed that the rhombic plate is subjected to a study one dimensional temperature distribution along the length, i.e. in x-direction,

\[ \tau = \tau_0 \left( 1 - \frac{x'}{a} \right) \quad (3) \]

Where \( \tau \) denotes the temperature excess above the reference temperature at any point at a distance \( \frac{x}{a} \) and \( \tau_0 \) denotes the temperature excess above the reference temperature at the end \( \frac{x}{a} = a \). The temperature dependence of the modulus of elasticity is given by

\[ E(\tau) = E_0 \left( 1 - \gamma \tau \right) \quad (4) \]

where \( E_0 \) is the Young modulus at the reference temperature i.e. at \( \tau = 0 \) with the temperature at the end of the plate as reference. The Young’s modulus in view of equation (3) and (4) become

\[ E(y') = E_0 \left( 1 - \alpha \left( 1 - \frac{x'}{a} \right) \right) \quad (5) \]

where \( \alpha = \gamma \tau_0 (0 \leq \alpha < 1) \)

The thickness variation of the plate is assumed to be linear in x-direction i.e.
In using the Rayleigh-Ritz technique, one requires that the maximum strain energy must be equal to the maximum kinetic energy. It is, therefore, necessary for the problem under consideration that
\[ \delta ( V - \lambda^2 T ) = 0 \] (7)

The maximum kinetic energy, V in the plate when it is executing transverse vibration mode shape \( W (x', y') \) are [1]:
\[ T = \frac{1}{2} \rho \omega^2 \cos \theta \iint h W^2 dx dy' \] (8)

And
\[ V = \frac{1}{2} \cos^3 \theta \iint D[W_{,xx}^2 - 4 \sin \theta W_{,xx} W_{,xy} + 2(\sin^2 \theta + \nu \cos^2 \theta) W_{,xx} W_{,xy}^2 + 2(1 + \sin^2 \theta - \nu \cos^2 \theta) W_{,xy}^2 - 4 \sin \theta W_{,xx} W_{,yy} W_{,xy}^2 - 4 \sin \theta W_{,xy} W_{,yy} W_{,yy}^2 ] dx dy' \] (9)

For arbitrary variations of W satisfying relevant geometric boundary conditions. For a rhombic plate clamped along all four edges the boundary conditions are
\[ W = W_{,xx} = 0 \text{ at } x' = 0, a \text{ and } W = W_{,yy} = 0 \text{ at } y' = 0, b \] (10)

And corresponding two term deflection function is taken as
\[ W(x', y') = \left( \frac{x^2}{a^2} \right) \left( \frac{y^2}{a^2} \right) \left[ A_1 + A_2 \left( \frac{x}{a} \right)^2 \left( \frac{y}{a} \right)^2 \left( 1 - \frac{x}{a} \right) \left( 1 - \frac{y}{a} \right) \right] \] (11)

Using eqn (8) & (9) in eqn (7) one gets
\[ \delta ( V^* - \lambda^2 T^* ) = 0, \ldots, n = 1, 2 \] (12)

where
\[ V^* = \frac{1}{\cos^3 \theta} \int_0^a \int_0^b \left[ (1 - \alpha(1 - \frac{x}{a})) (1 + \beta \frac{x}{a}) \right] W_{,xx}^2 - 4 \sin \theta W_{,xx} W_{,xy} + 2(\sin^2 \theta + \nu \cos^2 \theta) W_{,xx} W_{,xy}^2 + 2(1 + \sin^2 \theta - \nu \cos^2 \theta) W_{,xy}^2 - 4 \sin \theta W_{,xx} W_{,yy} W_{,xy}^2 - 4 \sin \theta W_{,xy} W_{,yy} W_{,yy}^2 ] dx dy' \]
\[ T^* = \frac{1}{2} \rho \omega^2 \int_0^a \int_0^b h W^2 dx dy' \]

and \( \lambda^2 = \frac{12a^4 \omega^2 \rho (1 - \nu^2)}{E_0 h^2} \)

Eqn (12) involves unknown constant \( A_1 \) & \( A_2 \) arising due to substitution of \( W(x', y') \) from eqn (11). These unknowns are to be determined from (1) for which
\[ \frac{\partial}{\partial A_n} ( V^* - \lambda^2 T^* ) = 0, \quad n = 1, 2 \] (13)

On simplifying one gets
\[ c_{n1} A_1 + c_{n2} A_2 = 0 \quad n = 1, 2 \] (14)

where \( c_{n1}, c_{n2} = c_{21}, c_{22} \) involve parametric constants and frequency parameter.

For a non-trivial solution the determinant of the coefficients of equation (14) must be zero i.e.
Frequency equation (15) is quadratic in $\lambda^2$, so it will give two roots. These two values represent the two modes of vibration of frequency i.e. $\lambda_1$ & $\lambda_2$ for different values of skew angle and thermal gradient. From equation (15) one can easily obtain frequency for both the mode.

### III. Result and Discussion

All computation has been done for frequencies of visco-elastic rhombic plate for different values of thermal gradient ($\alpha$), taper constant ($\beta$) and skew angle ($\theta$) for the first two modes of vibration.

**Table 1:** It is clearly seen that frequency decreases as the thermal gradient increases from 0.0 to 0.8 for $\beta$=0.0 and $\beta$=0.4 & $\theta$=30° for both modes of vibration.

**Table 2:** It is evident that value of frequency increases as the value of skew angle increases from 0° to 60° for $\beta$=0.0 and $\beta$=0.4 at fix $\alpha$ =0.0.

### IV. Conclusion

Motive is to provide such kind of a mathematical design so that scientist can perceive their potential in mechanical engineering field & increase strength, durability and efficiency of mechanical design and structuring with a practical approach. Actually this is the need of the hour to develop more but authentic mathematical model for the help of mechanical engineers/researchers/practitioners. Therefore mechanical engineers and technocrats are advised to study and get the practical importance of the present paper and to provide much better structure and machines with more safety and economy.

![Graph 1: Frequency vs thermal gradient with $\theta=30$](image1)

![Table 2: Frequency vs skew angle with $\alpha=0.0$](image2)

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<th>$\theta$</th>
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V. References


