INTRODUCTION OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

Generalized Beta Homeomorphisms in Intuitionistic Fuzzy Topological Spaces

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Abstract

In this paper we introduce the new class of homeomorphisms called generalized beta homeomorphisms in intuitionistic fuzzy topological spaces. We also introduce M-generalized beta homeomorphisms in intuitionistic fuzzy topological spaces and investigate some of the properties. We provide the relation between intuitionistic fuzzy generalized beta homeomorphisms and intuitionistic fuzzy M-generalized beta homeomorphisms. Also we prove that the set of all M-generalized beta homeomorphisms forms a group under the operation of composition of maps.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy generalized beta homeomorphisms and intuitionistic fuzzy M-generalized beta homeomorphisms.

I. Introduction

Zadeh [6] introduced fuzzy sets. After that Atanassov [1] introduced intuitionistic fuzzy sets. Using the notion of intuitionistic fuzzy sets, Coker [3] introduced the notion of intuitionistic fuzzy topological spaces. The notion of homeomorphisms plays a vital role in intuitionistic fuzzy topology as well as in topology. Here we introduce the new class of homeomorphisms called generalized beta homeomorphisms in intuitionistic fuzzy topological spaces. We also introduce the M-generalized beta homeomorphisms in intuitionistic fuzzy topological spaces and investigate some of the properties. We provide the relation between intuitionistic fuzzy generalized beta homeomorphisms and intuitionistic fuzzy M-generalized beta homeomorphisms. Also we prove that the set of all M-generalized beta homeomorphisms forms a group under the operation of composition of maps.

II. Preliminaries

Definition 2.1: [1] An intuitionistic fuzzy set (IFS in short) A in X is an object having the form

\[ A = \{ x, \mu_A(x), \nu_A(x), x \in X \} \]

where the functions \( \mu_A : X \to [0,1] \) and \( \nu_A : X \to [0,1] \) denote the degree of membership (namely \( \mu_A(x) \)) and the degree of non-membership (namely \( \nu_A(x) \)) of each element \( x \in X \) to the set \( A \), respectively, and \( 0 \leq \mu_A(x) + \nu_A(x) \leq 1 \) for each \( x \in X \). Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the form

\[ A = \{ x, \mu_A(x), \nu_A(x), x \in X \} \] and \[ B = \{ x, \mu_B(x), \nu_B(x), x \in X \} \]. Then

a) \( A \subseteq B \) if and only if \( \mu_A(x) \leq \mu_B(x) \) and \( \nu_A(x) \geq \nu_B(x) \) for all \( x \in X \)

b) \( A = B \) if and only if \( A \subseteq B \) and \( B \subseteq A \)

c) \( A^c = \{ x, \nu_A(x), \mu_A(x), x \in X \} \)

d) \( A \cap B = \{ x, \mu_A(x) \land \mu_B(x), \nu_A(x) \lor \nu_B(x), x \in X \} \)

e) \( A \cup B = \{ x, \mu_A(x) \lor \mu_B(x), \nu_A(x) \land \nu_B(x), x \in X \} \)

For the sake of simplicity, we shall use the notation \( A = \{ x, \mu_A(x), \nu_A(x), x \in X \} \) instead of \( A = \{ x, \mu_A(x), \nu_A(x), x \in X \} \). The intuitionistic fuzzy sets \( 0 = \{ x, 0, 1, x \in X \} \) and \( 1 = \{ x, 1, 0, x \in X \} \) are respectively the empty set and the whole set of X.


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Definition 2.3: [2] An intuitionistic fuzzy topology (IFT for short) on X is a family τ of IFSs in X satisfying the following axioms. 
(i) 0, 1 ∈ τ  
(ii) G₁ ∩ G₂ ∈ τ for any G₁, G₂ ∈ τ  
(iii) U ∪ Gᵢ ∈ τ for any family {Gᵢ / i ∈ J} ⊆ τ.
In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement Aᶜ of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [2] Let (X, τ) be an IFTS and A = (x, µₓ, νₓ) be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by
\[ \text{int}(A) = \bigcup \{ G \mid G \text{ is an IFOS in } X \text{ and } G \subseteq A \} \]
\[ \text{cl}(A) = \bigcap \{ K \mid K \text{ is an IFCS in } X \text{ and } A \subseteq K \} \]

Definition 2.5: [2] An IFS A = (x, µₓ, νₓ) in an IFTS (X, τ) is said to be an intuitionistic fuzzy beta open set (IFβOS for short) if \( \text{int}(\text{cl}(\text{int}(A))) \subseteq A \). An intuitionistic fuzzy beta closed set (IFβCS for short) if \( \text{cl}(\text{int}(\text{cl}(A))) \subseteq A \).

Definition 2.6: [3] Let A be an IFS in an IFTS (X, τ). Then the beta interior and the beta closure of A are defined by
\[ \beta\text{int}(A) = \bigcup \{ G \mid G \text{ is an IFβOS in } X \text{ and } G \subseteq A \} \]
\[ \beta\text{cl}(A) = \bigcap \{ K \mid K \text{ is an IFβCS in } X \text{ and } A \subseteq K \} \].
We have for any IFS A in (X, τ), \( \beta\text{cl}(A^\circ) = (\beta\text{int}(A))^\circ \) and \( \beta\text{int}(A^\circ) = (\beta\text{cl}(A))^\circ \) [3].

Definition 2.7: [3] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized beta closed set (IFβCS for short) if \( \beta\text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and U is an IFOS in (X, τ).
Every IFβCS is an IFGβCS but the converse may not be true in general [3].

Definition 2.8: [3] The complement Aᶜ of intuitionistic fuzzy generalized beta open set an IFGβCS A in an IFTS (X, τ) is called an (IFGβOS for short) in X.
Definition 2.9: [3] If every IFGβCS in (X, τ) is an IFβCS in (X, τ), then the space can be called as an intuitionistic fuzzy βT1/2 space (IFβT₁/₂ space for short).

Definition 2.10: [4] A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy generalized beta continuous mapping (IFGβ continuous mapping for short) if \( f^{-1}(V) \) is an IFGβCS in (X, τ) for every IFCS V of (Y, σ).

Definition 2.11: [4] A mapping f: (X, τ) → (Y, σ) is called intuitionistic fuzzy generalized beta irresolute (IFGβ irresolute) mapping if \( f^{-1}(V) \) is an IFGβCS in (X, τ) for every IFGβCS V of (Y, σ).

Definition 2.12: [5] A map f: X → Y is called an intuitionistic fuzzy generalized beta closed mapping (IFGβCM for short) if f(A) is an IFGβCS in Y for each IFCS A in X.

Definition 2.13: [5] A mapping f: X → Y is said to be an intuitionistic fuzzy generalized beta open mapping (IFGβOM for short) if f(A) is an IFGβOS in Y for each IFOS in X.

Definition 2.14: [5] A mapping f: X → Y is said to be an intuitionistic fuzzy M-generalized beta closed mapping (IFMβCM, for short) if f(A) is an IFGβCS in Y for every IFGβCS A in X.

### III. Generalized Beta Homeomorphisms in Intuitionistic Fuzzy Topological Spaces
In this section we introduce intuitionistic fuzzy generalized beta homeomorphisms and investigate some properties.

Definition 3.1: Let f: X → Y be a bijective mapping. Then f is said to be an intuitionistic fuzzy generalized beta homeomorphism (IFGβHM for short) if f is both an IFGβ continuous mapping and an IFGβOM.
Example 3.2: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \{x, (0.5, 0.6), (0.5, 0.4)\}$, $G_2 = \{y, (0.2, 0.3), (0.8, 0.7)\}$. Then $\tau = \{0.0, G_1, \ldots, \}$ and $\sigma = \{0.0, G_2, \ldots, \}$ are IFTs on $X$ and $Y$ respectively. Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then $f$ is an IFGHM.

Theorem 3.3: Let $f: X \to Y$ be a bijective mapping. If $f$ is an IFG continuous mapping, then the following are equivalent:

(i) $f$ is an IFGOM
(ii) $f$ is an IFGHM
(iii) $f$ is an IFGCM.

Proof: Straightforward.

Remark 3.4: The composition of two IFGHM need not be an IFGHM in general.

Example 3.5: Let $X = \{a, b\}$, $Y = \{c, d\}$ and $Z = \{e, f\}$. Let $G_1 = \{x, (0.5, 0.6), (0.5, 0.4)\}$, $G_2 = \{y, (0.8, 0.7), (0.2, 0.3)\}$, $G_3 = \{z, (0.8, 0.2), (0.2, 0.8)\}$ and Then $\tau = \{0.0, G_1, G_2, \ldots, \}$ and $\sigma = \{0.0, G_3, G_4, \ldots, \}$ are IFTs on $X$, $Y$ and $Z$ respectively. Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = c$ and $f(b) = d$ and $g: (Y, \sigma) \to (Z, \sigma)$ by $g(c) = d$ and $g(d) = e$. Then $f$ and $g$ are IFGHM but $g \circ f$ is not an IFGHM, since $g \circ f$ is not an IFG continuous mapping, since $G_1 = \{x, (0.6, 0.7), (0.4, 0.3)\}$ is an IFCS in $X$ but $(g \circ f)^{-1}(G_1) = \{x, (0.6, 0.7), (0.4, 0.3)\}$ is not an IFGCS in $X$, since $(g \circ f)^{-1}(G_1) = \{x, (0.6, 0.7), (0.4, 0.3)\} \subseteq G_2$ but $\mathcal{Bcl}(g \circ f)^{-1}(G_1) \neq 1 \notin G_2$.

Definition 3.6: Let $f: X \to Y$ be a bijective mapping. Then $f$ is said to be an intuitionistic fuzzy M generalized beta homeomorphism (IFMGHM for short) if $f$ is both an IFG irresolute mapping and an IFMGOM.

The family of all IFMGHM in $X$ is denoted by IFMGHM(X).

Theorem 3.7: Every IFGBM is an IFGHM but not conversely.

Proof: Let $f: X \to Y$ be an IFGBM. Let $A \subseteq Y$ be an IFCS. Then $A$ is an IFGCS in $Y$. By hypothesis, $f^{-1}(A)$ is an IFG continuous mapping. Let $B \subseteq X$ be an IFOS. Then $B$ is an IFGOS in $X$. By hypothesis, $f(B)$ is an IFGOS in $Y$. Hence $f$ is an IFGOM. Thus $f$ is an IFGHM.

Example 3.8: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \{x, (0.4, 0.6), (0.6, 0.4)\}$, $G_2 = \{y, (0.5, 0.7), (0.5, 0.3)\}$, $G_3 = \{z, (0.2, 0.3), (0.8, 0.7)\}$. Then $\tau = \{0.0, G_1, G_2, \ldots, \}$ and $\sigma = \{0.0, G_3, \ldots, \}$ are IFTs on $X$ and $Y$ respectively. Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then $f$ is an IFGHM but not an IFGBM, since $A = \{y, (0.4, 0.7), (0.6, 0.3)\}$ is an IFCS in $Y$ but $f^{-1}(A)$ is not an IFGCS in $X$, since $f^{-1}(A) = \{x, (0.4, 0.7), (0.6, 0.3)\} \subseteq G_2$ but $\mathcal{Bcl}(f^{-1}(A)) = 1 \notin G_2$.

Theorem 3.9: The composition of two IFGHM is an IFGHM.

Proof: Let $f: X \to Y$ and $g: Y \to Z$ be any two IFGSPHMs. Let $A \subseteq Z$ be an IFGCS. Then by hypothesis, $g^{-1}(A)$ is an IFGCS in $Y$. Again by hypothesis, $f^{-1}(g^{-1}(A))$ is an IFGCS in $X$. Therefore $g \circ f$ is an IFG irresolute mapping. Now let $B \subseteq X$ be an IFGOS. Then by hypothesis, $f(B)$ is an IFGOS in $Y$ and also $g(f(B))$ is an IFGOS in $Z$. This implies $g \circ f$ is an IFGOM. Hence $g \circ f$ is an IFGHM. Therefore $g \circ f$ is an IFGHM.

Theorem 3.10: Let $f: X \to Y$ be a bijective mapping. If $f$ is an IFG irresolute mapping, then the following are equivalent:

(i) $f$ is an IFGOM
(ii) $f$ is an IFGHM
(iii) $f$ is an IFGCM.

Proof: Straightforward.

Theorem 3.11: The set of all IFMGHM in an IFTS $(X, \tau)$ is a group under the composition of maps.

Proof: Define a binary operation $* : \text{IFMGHM}(X) \times \text{IFMGHM}(X) \to \text{IFMGHM}(X)$ by $f \ast g = g \circ f$ for every $f, g$. Submitted by the authors to the International Journal of Engineering Sciences & Research Technology [230-233].
Theorem 3.12: If \( f : X \to Y \) is an IFMG\( \beta \)HM, then \( g\beta cl(f^{-1}(B)) \subseteq f^{-1}(\beta cl(B)) \) for every IFS \( B \) in \( Y \).

Proof: Let \( B \subseteq Y \). Then \( \beta cl(B) \) is an IFG\( \beta \)CS in \( Y \). Since \( f \) is an IFG\( \beta \) irresolute mapping, \( f^{-1}(\beta cl(B)) \) is an IFG\( \beta \)CS in \( X \). This implies \( g\beta cl(f^{-1}(\beta cl(B))) \subseteq f^{-1}(\beta cl(B)) \) Now \( g\beta cl(f^{-1}(B)) \subseteq g\beta cl(f^{-1}(\beta cl(B))) \) \( = f^{-1}(\beta cl(B)) \).

Theorem 3.13: If \( f : X \to Y \) is an IFMG\( \beta \)HM, where \( X \) and \( Y \) are IF\( \beta \)T\(_{1/2}\) spaces, then \( \beta cl(f^{-1}(B)) = f^{-1}(\beta cl(B)) \) for every IFS \( B \) in \( Y \).

Proof: Since \( f \) is an IFMG\( \beta \)HM, \( f \) is an IFG\( \beta \) irresolute mapping. Since \( \beta cl(f(B)) \) is an IFG\( \beta \)CS in \( Y \), \( f^{-1}(\beta cl(f(B))) \) is an IFG\( \beta \)CS in \( X \). Since \( X \) is an IF\( \beta \)T\(_{1/2}\) space, \( f^{-1}(\beta cl(f(B))) \subseteq \beta cl(f^{-1}(B)) \). Now \( f^{-1}(\beta cl(B)) \subseteq f^{-1}(\beta cl(f^{-1}(B))) \) \( = f^{-1}(\beta cl(B)) \). This implies \( \beta cl(f^{-1}(B)) \subseteq f^{-1}(\beta cl(B)) \) \( \subseteq f^{-1}(f^{-1}(\beta cl(B))) \) \( = f^{-1}(\beta cl(B)) \) \( \subseteq f^{-1}(\beta cl(f^{-1}(B))) \) \( = f^{-1}(\beta cl(B)) \) \( = f^{-1}(\beta cl(f^{-1}(B))) \) \( = f^{-1}(\beta cl(B)) \).

Corollary 3.14: If \( f : X \to Y \) is an IFMG\( \beta \)HM, where \( X \) and \( Y \) are IF\( \beta \)T\(_{1/2}\) spaces, then \( \beta cl(f(B)) = f(\beta cl(B)) \) for every IFS \( B \) in \( X \).

Proof: Since \( f \) is an IFMG\( \beta \)HM, \( f^{-1} \) is also an IFMG\( \beta \)HM. Therefore by Theorem 3.13 \( \beta cl(f^{-1}(B)) = f^{-1}(\beta cl(B)) \) for every IFS \( B \) in \( X \).

Corollary 3.15: If \( f : X \to Y \) is an IFMG\( \beta \)HM, where \( X \) and \( Y \) are IF\( \beta \)T\(_{1/2}\) spaces, then \( \beta int(f(B)) = f(\beta int(B)) \) for every IFS \( B \) in \( X \).

Proof: For any IFS \( B \subseteq X \), \( \beta int(B) = (\beta cl(B^c))^c \). By Corollary 3.14, \( f(\beta int(B)) = f(\beta cl(B^c))^c \) \( = (f(\beta cl(B^c))^c \) \( = (\beta cl(f(B^c))^c \) \( = \beta int(f(B^c))^c \) \( = \beta int(f(B)) \).

Corollary 3.16: If \( f : X \to Y \) is an IFMG\( \beta \)HM, where \( X \) and \( Y \) are IF\( \beta \)T\(_{1/2}\) spaces, then \( \beta int(f^{-1}(B)) = f^{-1}(\beta int(B)) \) for every IFS \( B \) in \( Y \).

Proof: Since \( f \) is an IFMG\( \beta \)HM, \( f^{-1} \) is also an IFMG\( \beta \)HM, the proof directly follows from Corollary 3.15.

IV. References