Experimental and Theoretical Investigation Into Rotary Forging of Axi-Symmetric Aluminium Disc

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Abstract

The present paper focuses on the investigation into rotary forging of axi-symmetric aluminium disc experimentally and ‘Upper Bound’ theoretical analysis. Conical die with an included angle of 170°, i.e. skew angle of 5° was fabricated and experiments were performed using these dies on radial drilling machines. The deformation and conical indented contact area were critically investigated. The theoretical analysis was done based on ‘Upper Bound’ approach, where exponential velocity field and corresponding strain rates satisfying necessary boundary conditions were proposed. Using these expressions, total energy expended during rotary forging was computed, which was used to compute the average rotary forging load. For ease of computation mechanics, the entire rotary forging process was considered into two phases, i.e., indentation phase and rotary phase. The final average rotary forging load was computed by summing up the average loads during indentation and rotary phases. The effect of various rotary forging characteristics like axial feed rate, rotational speed, skew angle on percent deformation, average forging load etc were studied. It is expected that present work will be useful for the prediction of the rotary forging load, as well for the design of rotary forging processes.

Keywords: deformation & conical indented contact area, total energy, average rotary forging load

Introduction

Theoretical research plays a very important role in the development and spared of a technology. However, because the rotary forging process shows a fairly complicated deformation, it is very difficult to obtain satisfactory result. In the recent years, several attempts have been made to analyze the rotary forging process by the use of the “Upper Bound” method. Su-Hai Hsiang, Huey-Lin Ho.[1] A Study on the warping problems of thin flange under the radial forging processes by FEM and Experiments. The radial forging process of work-hardened materials is simulated, the phenomenon of flange warping is analyzed, and the accuracy of the simulation is experimentally verified. B. Pan, D.J. Kim, B.M. Kim, T.A. Dean.[2] Incremental deformation and the forgeability of titanium aluminid. The effect of various forms of incremental deformation, with or without subsequent isothermal forging, on microstructure is discussed. A. Ghaei, M.R. Movahhedy, A. Karimi Taheri.[3] Study of the effects of die geometry on deformation in the radial forging process. This process is usually used for reducing the diameters of ingots and bars, forging of stepped shafts and axels, forging of rifle barrels and for production of tubular components with and without internal profiles. Wang Guangchun, Guan Jing, Zhao Guoqun.[4] A photo-plastic experimental study on deformation of rotary forging a ring workpiece. The basic principle of rotary forging, the basic theory of photo-plastic experimental methods, and the mechanical and optical properties of photo-plastic material were introduced briefly. The heat treatment process of photo-plastic material was designed and made. The photo-plastic experiments of rotary forging a ring workpiece were acted by two schemes using polycarbonate samples at a 300 kN rotary forging press. Through taking photographs of equi-different charts and drawing isoline charts of slices at the exit and entrance of the contact zone of samples, the strain distributions of crossing line of slices were calculated by the oblique passing method. G. Liu, S.J. Yuan, Z.R. Wang, D.C. Zhou.[5]Explanation of the mushroom effect in the rotary forging of a cylinder. JH Park, YH Kim, YE Jain.[6] Experimental Investigation of forming parameters of the rotational upset forging process. In this study to solve the above problem, a parametric study is presented a new rotational upset forging process, twisting torque can change the harmful effect of friction into a...
useful effect and experiments were carried out to generate data of forming parameters which are forming factor, friction factor, sliding factor, the punch velocity and geometry of the billet on the rotational upset forging process. S. Choi, K.H. N, J.H. Kim. [7] Upper-bound analysis of the rotary forging of a cylindrical billet. For the analysis of this forging process, a simple velocity field which can satisfy all of the velocity boundary conditions is proposed. From the derived velocity field the upper-bound force is determined by minimizing the total power consumption with respect to some chosen parameters, and velocity distribution is investigated also. Experiments are carried out with carbon steel (AISI1020, AISI1045) and aluminium (6061) and the results then compared with theoretical results. Hung-Kuk Oh, Seogou Cho. [8] A study on centre thinning in the rotary forging of a circular plate. This paper intends to explain the phenomenon of center thinning and gives a criterion of it. In order to confirm the validity of the proposed criterion, experiments have been carried out using a rotary forging press that has been designed and constructed in the authors’ laboratory. 6 1997 Elsevier Science S.A.

**Experimental Investigation**

The experimental investigations during rotary forging of axi-symmetric aluminium disc consists of fabrication and assembly of rotary forging die set, fabrication / machining of workpiece into required dimensions and performing the requisite forging process on drilling machine.

**Fabrication of Rotary Forging Dies:**

The rotary forging die has been specially designed in composite nature, where two separate parts are fabricated and assembled together to manufacture the complete die set using Harden Steel (En24) material as shown in figure 2.1. This is done for the ease of fabrication and fool proof inclination of the conical front end of the die, so that one of its generators is parallel to the horizontal axis of the forging machine table. Any deviation from this will lead to defects in the final products during rotary forging and will lead to skewed top surface on the final forged product. Figure 2.2 shows the representation of the design of front conical tool of the rotary forging die. The front end is machined in conical shape with requisite included angle of 170°, i.e. skew contact angle of 5° as shown in figure 2.3(a). The die tool holder is fabricated separately from the same material and shown in figure 2.3(b), which is assembled at the back of the conical front with screw arrangement. The complete assembled rotary forging tool is shown in figure 2.4. The back end of the tool holder has been machined in the form of tool shank, which goes and fits inside the tool holder spindle of the radial drilling machine.
Description of Rotary Forging Process

The chemical compositions of the rotary forging die material Hardened Steel (En24) as provided by the manufacturer is mentioned in table 2.1. As per the specifications, the die material is heat treated by heating it uniformly to about 850°C then quenched in water to increase its hardness and resistance against wear & tear. The workpiece material considered for the present research work of rotary forging of axi-symmetric disc has been fabricated from the pure aluminium material. The purpose of choosing pure aluminium was due to the distinct advantage of it being highly corrosion resistant, making the products useful under corrosive environment also.

Table 2.1  Composition of Rotary Forging Die En24

<table>
<thead>
<tr>
<th>Elements</th>
<th>C.</th>
<th>Si.</th>
<th>Mn</th>
<th>Ni</th>
<th>Cr</th>
<th>Mo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent</td>
<td>0.40%</td>
<td>0.30%</td>
<td>0.60%</td>
<td>1.50%</td>
<td>1.20%</td>
<td>0.25%</td>
</tr>
</tbody>
</table>

The rotary forging experiments were carried by holding the axi-symmetric aluminium disc within the vice mounted on the table of drilling machine as shown in figure 2.5. The rigidity of the workpiece was ensured properly before the commencement of the experiments within the workshop. After the workpiece was firmly clamped on the table, the rotary forging die set with receding tool holder shank was inserted inside the spindle of the tool holder of the drilling machine as shown in the figure 2.6. The required axial feed was provided by moving the table upwards and the tool spindle was rotated at requisite speed to provide the rotational speed to the rotary forging die set.

During rotary forging process, the indentation phase was performed initially by axial feed of upper conical die into the top surface of workpiece, which is followed by sweeping the indented area throughout the preform top surface by angular rotation of the upper conical die at room temperature under dry lubricating conditions. Figure 2.7 shows the illustration of the deformed axi-symmetric aluminium disc after rotary forging process. Table 2.2 shows the typical data for the rotary forging process performed in the research investigations.
Theoretical Analysis

The present research problem considers rotary forging of axi-symmetric Aluminium disc at cold conditions between a fixed flat bottom die and a rotating conical upper die with inclined cone axis. The present theoretical analysis considers following assumptions:

a. The complete rotary forging process is analyzed into two subsequent separate phases, namely indentation and rotary phases; thereafter results are combined to get the final load.

b. The bulging of vertical faces of axi-symmetric disc has been incorporated by considering barreling factor in the exponential velocity field.

c. The interfacial friction considered is composite in nature, which includes sliding, as well as sticking frictions, where sticking friction is considered as a function of an adhesion factor, which is a material property.

d. The radial flow is not uniform throughout the circumference of indented area and it is maximum at central axis of symmetry and gradually decrease and become nil at the ends. This leads to circumferential material flow (i.e. $U_θ ≠ 0$).

Analysis of Indented Contact Area: Figure 3.1 shows the schematic representation of rotary forging process using upper conical and flat lower dies along with the coordinate axes. It may be noted that the cone axis of upper conical die is swelled by an angle ‘α’ with respect to vertical axis, which is the skew contact angle of the upper conical die. During indentation phase, the upper conical die is fed axially into the axi-symmetric aluminium disc top surface upto the required indentation depth. Figure 3.2 shows the projection of the conical indented contact on the ‘XOY’ plane (only half area has been shown due to its symmetry). The equations for conical surface of upper conical die and flat top surface of axi-symmetric aluminium workpiece are given as:

$$x^2 + y^2 = \left(\frac{z^2}{\tan^2 \alpha}\right) = 0 \quad (3.1)$$

$$\left(\frac{z \cos \alpha}{h_i}\right) - \left(\frac{x \tan \alpha}{h_i \cos \alpha}\right)^2 = 1 \quad (3.2)$$

Equation for projected conical indented surface created on the top surface of axi-symmetric aluminium disc (i.e. for curve ‘AEC’ in ‘XOY’ plane) is formulated by solving equations (3.1) and (3.2) simultaneously, which is given as (neglecting higher order terms):

$$y = \left(\frac{h_i}{\tan \alpha}\right)\left[\left(\frac{2x \tan \alpha}{h_i}\right) + 1\right]^{\frac{1}{2}} \quad (3.3)$$

The equation of top surface of axi-symmetric aluminium disc in ‘XOY’ plane is a circle and may be expressed as:

$$x^2 + y^2 = R_0^2 \quad (3.4)$$
where

\[
(3.7)
\]

Thus, the indented contact area on top surface of a \(xi\)-contact area may be given as:

\[
\text{Integrating equation (3.7) and simplifying, the indented contact area on top surface of a xi-contact area may be given as:}
\]

\[
(3.8)
\]

Hence, Indented Contact Area Rate may be expressed as:

\[
\text{ICR} = \left( \frac{A_{av}}{\pi R_i^2} \right) \quad (3.9)
\]

### 3.2 Analysis of Indentation Phase

The compatibility equation (by volume constancy principal) for deformation of metals is given as follows:

\[
\epsilon_{rr} + \epsilon_{\theta\theta} + \epsilon_{zz} = 0 \quad (3.10)
\]

The boundary conditions during indentation phase of rotary forging are given as follows:

\[
U_z = 0 \text{ at } z = h, \quad U = 0 \text{ at } z = h_1 \quad \text{and } U_0 = 0 \text{ at } r = 0 \quad (3.11)
\]

The velocity field and strain rates satisfying equations (3.10) & (3.11) are given as follows [refer appendix]:

\[
U_r = \beta e^{\beta h} U(1 + \cos \theta h) \left( \frac{1}{2} \right) \quad (3.12)
\]

\[
U_\theta = \beta e^{\beta h} U \sin \theta h \quad (3.13)
\]

\[
U_z = \left( 1 - e^{\beta h} \right) U \quad (3.14)
\]

\[
\dot{\epsilon}_{rr} = \frac{\partial U_r}{\partial r} = \beta e^{\beta h} U(1 + \cos \theta h) \left( \frac{1}{2} \right) \quad (3.15)
\]

\[
\dot{\epsilon}_{\theta\theta} = \frac{U_r + \partial U_\theta}{r \partial \theta} = \beta e^{\beta h} U(1 - \cos \theta h) \left( \frac{1}{2} \right) \quad (3.16)
\]

\[
\dot{\epsilon}_{zz} = \frac{\partial U_z}{\partial z} = \beta e^{\beta h} U \quad (3.17)
\]

\[
\dot{\epsilon}_{\theta\theta} = \frac{1}{2} \left[ \frac{\partial U_z}{\partial r} + \frac{\partial U_\theta}{\partial r} - \frac{U_z}{r} \right] = \frac{\beta e^{\beta h} U \cos \theta h}{4(1 - e^{\beta h})} \quad (3.18)
\]

\[
\dot{\epsilon}_{zz} = \frac{1}{2} \left[ \frac{\partial U_z}{\partial z} + \frac{\partial U_\theta}{\partial r} \right] = \frac{\beta e^{\beta h} U \sin \theta h}{P(1 - e^{\beta h})^2} \quad (3.19)
\]

\[
\dot{\epsilon}_{rr} = \frac{1}{2} \left[ \frac{\partial U_r}{\partial r} + \frac{\partial U_\theta}{\partial r} - \frac{U_r}{r} \right] = \frac{\beta e^{\beta h} U(1 + \cos \theta h)}{4(1 - e^{\beta h})^2} \quad (3.20)
\]
The total external power ‘J’ supplied by the die platens during rotary forging using ‘Upper Bound’ approach is given as follows:

\[ J = W_i + W_f = \frac{2\sigma_0}{\sqrt{3}} \int \frac{1}{2} \hat{e} \hat{\epsilon}_0 \, dV + \int \frac{d\Delta U}{\Delta S} \]  

(3.21)

The first term on the right side denotes internal energy dissipation ‘\(W_i\)’, and the second term denotes frictional shear energy dissipation ‘\(W_f\)’. The energy dissipation have been computed by integrating the respective expressions into two parts, namely, from \(\theta = 0\) to ‘\(\alpha\)’ and \(\theta = \alpha\) to ‘\(\pi\)’. The corresponding expressions for radial distance are \(r = R_0\) and \(r = \{h_i / [\tan \alpha (1+\cos \Theta)]\}\) respectively. The internal energy dissipation ‘\(W_i\)’ may be computed as follows:

\[ W_i = \left( \frac{\sigma_0 U h_i^2}{96 \tan \alpha} \right) + \left( \frac{4 + \beta^2}{3} \right) \frac{8 \tan \alpha}{8 \tan \alpha} \left( \tan \frac{\pi P}{2} + 3 \tan \frac{\pi P}{2} + 4 \right) \]

(3.22)

Substituting equations (3.15) to (3.20) above, solving and simplifying, the internal energy dissipation may be expressed as follows:

\[ W_i = \left( \frac{\sigma_0 U h_i^2}{96 \tan \alpha} \right) + \left( \frac{4 + \beta^2}{3} \right) \frac{8 \tan \alpha}{8 \tan \alpha} \left( \tan \frac{\pi P}{2} + 3 \tan \frac{\pi P}{2} + 4 \right) \]

(3.32)

The boundary conditions during rotary phase of forging are given as follows:

\[ U_r = 0 \text{ at } z = 0; \quad U_r = -\cos \alpha \text{ at } z = h_i; \quad U_\theta = 0 \text{ at } r = 0 \text{ and } \theta = \alpha \]  

(3.29)

The velocity field and strain rates satisfying equations (3.10) & (3.29) is given as follows:

\[ U_r = \frac{\beta e^{-\beta z / h_i} r \cos \alpha (1 + \cos \Theta)}{2 (1 - e^{-\beta}) h_i} \]

(3.30)

\[ U_\theta = \frac{\beta e^{-\beta z / h_i} r \cos \alpha \sin \Theta}{P (1 - e^{-\beta}) h_i} \]

(3.31)

\[ U_z = \frac{(1 - e^{-\beta z / h_i}) r \cos \alpha}{1 - e^{-\beta}} \]

(3.32)

\[ \dot{e}_r = \frac{\partial U_r}{\partial r} = \frac{\beta e^{-\beta z / h_i} r \cos \alpha (1 + \cos \Theta)}{(1 - e^{-\beta}) h_i} \]

(3.33)

\[ \dot{e}_\theta = \frac{U_r + \frac{1}{r} \partial U_r}{2 \theta} = \frac{(1 - 2\theta) \beta e^{-\beta z / h_i} r \cos \alpha (1 - \cos \Theta) h_i}{2 \theta (1 + \theta)(1 - e^{-\beta}) h_i} \]

(3.34)

\[ \dot{e}_z = \frac{\partial U_z}{\partial Z} = \frac{-\beta e^{-\beta z / h_i} r \cos \alpha}{(1 - e^{-\beta}) h_i} \]

(3.35)

\[ \dot{e}_a = \frac{1}{2} \frac{\partial^2 U_a}{\partial \theta^2} + \frac{U_r}{r} = \frac{\beta e^{-\beta z / h_i} r \cos \alpha \sin \Theta}{4 (1 - e^{-\beta}) h_i} \]

(3.36)
The internal energy dissipation \( W_i \) during rotary phase of rotary forging may be computed as follows:

\[
W_i = \left( \sigma_\alpha \right)^2 \left( \frac{\dot{\varepsilon}_y \dot{\varepsilon}_y}{2} \right) dv
\]

Substituting equations (3.33) to (3.38) above, solving and simplifying, the internal energy dissipation may be expressed as follows:

\[
W_i = \frac{2\sigma_\alpha \sin \theta \sin \phi}{1 + \frac{3}{4} \tan \alpha (1 + \cos \theta \theta \theta \theta \beta)} \left( \frac{\dot{\varepsilon}_y \dot{\varepsilon}_y}{2} \right) (rdrd \alpha dz)
\]

(3.39)

The frictional shear energy dissipation at die-workpiece interface \( W_i' \) during rotary phase of rotary forging may be computed as follows:

\[
W_i' = \int_0^\pi \int_0^{\theta=\alpha} \frac{h_0}{\tan \alpha (1 + \cos \theta \theta \theta \beta)} \Delta U \, dr \, d\theta
\]

(3.40)

The frictional shear stress during rotary forging at \( z = h \) may be computed by equation (3.25) and the resultant magnitude of velocity along the direction of frictional shear stress during rotary forging is computed by substituting equations (3.30) & (3.32) into equation (3.26), which is as follows:

\[
\left( \dot{U}_i + \dot{U}_z \right)_{h} = \left[ \frac{\beta e^{-\theta \theta \theta \beta}}{1 - e^{-2\theta \theta \beta}} \right] \left[ \left( \frac{\cos \theta \theta \theta \beta}{2} \right) + \left( \frac{\sin \theta \theta \theta \beta}{P} \right) \right]^{\frac{1}{2}}
\]

(3.42)

Substituting equations (3.25) and (3.42) into equation (3.41), solving and simplifying, the frictional shear energy dissipation may be expressed as:
times for rotary forging process, especially at higher die velocities.

Figure 3.3 Variation of average indentation load with skew contact angle of upper conical die

Figure 3.4 Variation of average indentation load with indentation depth

The figure 3.4 shows the variations of average indentation load with indentation depth for different die velocity. It is clearly observed that indentation load increases with indentation depth, as higher loads are required to penetrate the top surface of axi-symmetric aluminium disc. Also, the indentation loads are lower for higher axial die velocity of upper conical die, as discussed above. The variation of load factor $\zeta$ with angular die velocity of upper conical die during rotary forging of axi-symmetric aluminium workpiece is shown in figure 3.5. It can be observed that die load decreases exponentially with increase in angular velocity of upper conical die and becomes asymptote to x-axis at higher angular die velocities. It shows that dynamic effects are predominant at higher deformation speeds, as compared with those at rotary forging at slower speed due to smaller contact times under loads. Also, the higher skew contact angles of the dies results into lower magnitude of load factors, which shows that higher skew contact angles leading to lower contact areas results into lower forging loads. The short forging times results into decrease in resistance of workpiece material against deformation and hence lower die loads.

Figure 3.5 Variation of load factor with angular velocity of upper conical die

Figures 3.6 shows the variation of height reduction with angular velocity of upper conical die for different average rotary load during rotary forging of axi-symmetric aluminium workpiece. It can be seen that reduction in height increases rapidly with the angular die velocity and then remains fairly constant. Also, it is higher for higher rotary loads and initial indentation depths. Figure 3.7 shows the variation of average rotary load with skew contact angle of the upper conical die for different angular velocities during rotary forging of axi-symmetric aluminium workpiece. It is evident from the figure that rotary loads are lower for higher skew contact angle and lower indentation depth indicating that load requirements with upper conical die having smaller cone angle is comparatively low as compared that with higher cone angle during rotary forging process. The final rotary forging loads also decreases with increase in angular velocity of die. This appears to be mainly of two prominent consequences. First, rotary forging, especially at higher die velocity is characterized by very small deformation contact time resulting into restriction of heat dissipation at die-workpiece interface and thus, reduction in the resistance of workpiece material against deformation. Second, the rotary forging produces an additional horizontal component of circumferential stress due to rotation or dynamic effects of upper conical die, which not only creates conditions necessary for better metal flow, but also reduces the net resultant stress required for deformation of sintered material. This effect also increases with angular velocity.
of upper conical die. This confirms the advantage of high load reduction offered by the rotary forging with conical dies over that of conventional flat-die forging operations. Also, the experimental data are in close agreement with theoretical ones and thus, confirms the validity of the present theoretical ‘Upper Bound’ analysis.

- The effect of skew contact angle of the conical upper die on forging load was studied and it was found that increase in skew contact angle leads to decrease in the die load, which is due to smaller indented contact areas with die having low skew contact angles. Also, the load decreases with increase in the axial feed rate, i.e. lower die vertical velocity and rotational speed of the upper conical die. This is attributed due to the inertia effects and the probable reason is that higher die speeds leads to heating of the axi-symmetric disc work piece, which ultimately may be reducing the yield stress of the material.

- The effect of the angular velocity of the upper conical die on the work piece height reduction at various die loads was investigated and it was found that percent height reduction of the axi-symmetric aluminium disc increases with increase in die velocity at higher forging loads. The trend of the graphs shows that this increase in the percent height reduction is predominant during the initial stages of the deformation and it remains fairly constant during the later stages of the deformation. This may be attributed due to increase in the resistance for deformation during latter stages of the deformation by the work piece material due to strain hardening.

- The inertia effect was clearly demonstrated using “Inertia Factor” and it was defined as the ratio of the forging load with inertia effects (U=0.1m/s) and without inertia effects (U=0.001m/s), the decrease in load factor with increase in rotational speed of the upper conical die and lower skew contact angles of the dies confirms that load requirements are low at high speed of deformation. This is due to the fact that heat generated during the high-speed rotary forging does not escape out rapidly from the work piece and actually affects the yield stress of the material by bringing its magnitude down during the forging process. Though, extensive investigations are required in this direction to confirm and validate the findings.

- Rotary forging process is comparatively advantageous to conventional flat die forging process due to smaller indented contact area, which leads to smaller deformation forging loads at any instant of time. Thus, the load transmitted to die platens and forging equipments is comparatively low as compared to similar flat die forging case and hence the tooling & equipment life will be more. Also, the low forging load requirements will lead to smaller sizes of the die sets and forging equipments leading to material and energy savings. This will lead to cost reduction in the manufacturing of the components made from rotary forging process.

Conclusions

The major conclusions drawn from the present research work are as follows:

- The flow behavior of axi-symmetric aluminium disc material during rotary forging process has been critically investigated. It was found that axial feed, i.e., die velocity in vertical direction leads to indented contact area on the work piece top surface and the size of the indented contact area grows with the indentation depth. Also, larger indentation load will required to create larger indented contact area. This effect is predominant with higher axial feed rate and higher rotational speed of the die sets. Also, the formulation of indented contact area has been mathematically demonstrated under the theoretical analysis, which is the corner stone for the “Upper Bound” analysis.

- The inertia effect was clearly demonstrated using “Inertia Factor” and it was defined as the ratio of the forging load with inertia effects (U=0.1m/s) and without inertia effects (U=0.001m/s), the decrease in load factor with increase in rotational speed of the upper conical die and lower skew contact angles of the dies confirms that load requirements are low at high speed of deformation. This is due to the fact that heat generated during the high-speed rotary forging does not escape out rapidly from the work piece and actually affects the yield stress of the material by bringing its magnitude down during the forging process. Though, extensive investigations are required in this direction to confirm and validate the findings.

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[347-356]
• The rotary motion of the conical dies and its precise control is very important as far as the quality of the forged components is considered and hence, extra precautions must be taken in order to design and execute the present forging process. It may be assumed that the present research work though is a preliminary finding in the area of rotary forging but still gives an insight into the present forging process and may be useful for the researchers to carry forward the work in the area of rotary forging.

References