ABSTRACT

This paper is devoted to fractional q-derivative of special functions. To begin with the theorem on term by term q-fractional differentiation has been derived. fractional q-differentiation of M- function [9] has been obtained.

KEYWORDS AND PHRASES: Fractional integral and derivative operators, Fractional q-derivative, M- function[9] and Special functions.

Mathematics Subject Classification: Primary33A30, Secondary 33A25, 83C99

INTRODUCTION

DEFINITION

q-Analogue of Differential Operator

Al-Salam [3], has given the q-analogue of differential operator as

\[ D_q f(x) = \frac{f(xq) - f(x)}{x(q - 1)} \]  \hspace{1cm} (1.1)

This is an inverse of the q-integral operator defined as

\[ \int_x^\infty f(t) d(t; q) = x(1 - q) \sum_{k=1}^{\infty} q^{-k} f(xq^{-k}) \] \hspace{1cm} (1.2)

WHERE \( 0 < |q| < 1 \)

FRACTIONAL Q-DERIVATIVE OF ORDER \( \alpha \)

The fractional q-derivative of order \( \alpha \) is defined as

\[ D_{x,q}^\alpha f(x) = \frac{1}{\Gamma_q(-\alpha)} \int_0^x (x - yq)^{-\alpha-1} f(y) d(y; q) \] \hspace{1cm} (1.2.1)

WHERE \( \text{Re}(\alpha) < 0 \)

As a particular case of (1.2.1), we have

\[ D_{x,q}^\alpha x^{\mu-1} = \frac{\Gamma_q(\mu)}{\Gamma_q(\mu - \alpha)} x^{\mu-\alpha-1} \] \hspace{1cm} (1.2.2)

M – function [9] –
We give the new special function, called $M_\alpha function [9] ^\mu \gamma_\delta \rho \ k_1 \ldots k_p \Gamma_1 \ldots \Gamma_q \ z \ \beta$ which is the most generalization of New Generalized Mittag-Leffler Function. Here, we give first the notation and the definition of the New Special $M_\alpha function [9]$, introduced by the author as follows:

$$\alpha, \beta, \gamma, \delta, \rho \ \under{\sum_{n=0}^\infty} (a_1)_n \ldots (a_p)_n (y)_n (\delta) \ n \ k_1^n \ldots k_p^n \ (ct)^{(n+y)}\alpha-\beta-1 \ \ (1,2,3)$$

There are $p$ upper parameters $a_1, a_2, \ldots, a_p$ and $q$ lower parameters $b_1, b_2, \ldots, b_q, \alpha, \beta, \gamma, \delta, \rho \in \mathbb{C}, Re(\alpha) > 0, Re(\beta) > 0, Re(\gamma) > 0, Re(\delta) > 0, Re(\rho) > 0, Re(\alpha \gamma - \beta) > 0$ and $(a_k)_k$ is the pochhammer symbols and $k_1, \ldots, k_p, l_1, \ldots, l_q$ are constants. The function (1) is defined when none of the denominator parameters $b_j, j = 1, 2, \ldots, q$ is a negative integer or zero. If any parameter $a_j$ is negative then the function (1) terminates into a polynomial in $(t)$.

**MAIN RESULTS**

In this section we drive the results on term by term $Q$-fractional differentiation of a power series. As particular Case we will the fractional $Q$-differentiation of the $M_\alpha function$.

**THEOREM 1**: If the series $\under{\sum_{n=0}^\infty} (a_1)_n \ldots (a_p)_n (y)_n (\delta) \ n \ k_1^n \ldots k_p^n \ (ct)^{(n+y)}\alpha-\beta-1$ converges absolutely for $|q| < \rho$ then

$$D_z^\mu \ z^{\alpha-1} \ \under{\sum_{n=0}^\infty} (a_1)_n \ldots (a_p)_n (y)_n (\delta) \ n \ k_1^n \ldots k_p^n \ (ct)^{(n+y)}\alpha-\beta-1 \ \ (2.1)$$

Where $Re(\alpha) > 0, Re(\mu) < 0, 0 < |q| < 1$

**PROOF**: Starting From the left side and using equation (1.2,1), we have

$$D_z^\mu \ z^{\alpha-1} \ \under{\sum_{n=0}^\infty} (a_1)_n \ldots (a_p)_n (y)_n (\delta) \ n \ k_1^n \ldots k_p^n \ (ct)^{(n+y)}\alpha-\beta-1 \ \ (2.2)$$

Now the following observation are made

(i) $$\under{\sum_{n=0}^\infty} (a_1)_n \ldots (a_p)_n (y)_n (\delta) \ n \ k_1^n \ldots k_p^n \ (ct)^{(n+y)}\alpha-\beta-1 \ \ z^k$$

converges absolutely and therefore uniformly on domain of $x$ over the region of integration.

(ii) $$\int_0^1 [1 - t^q]_{-1}^z \ d(t; q)$$

is convergent,

Provided $Re(\alpha) > 0, Re(\mu) < 0, 0 < |q| < 1$

Therefore the order of integration and summation can be interchanged in (2.2) to obtain,

\[
\frac{z^{\lambda-\mu-1}}{\Gamma_q(-\mu)} \sum_{n=0}^{\infty} \frac{(a_1)_n \cdots (a_p)_n (\gamma)_n (\delta)_n k_1^{-n} \cdots k_p^{-n}}{(b_1)_n \cdots (b_q)_n (\rho)_n l_1^{-n} \cdots l_q^{-n}} \frac{(cz)^{(n+\gamma)\alpha-\beta-1}}{n! \Gamma((n + \gamma)\alpha - \beta)} \int_0^1 (1 - tq)^{-n-1} t^{k-1} d(t; q)
\]

\[
= \sum_{n=0}^{\infty} \frac{(a_1)_n \cdots (a_p)_n (\gamma)_n (\delta)_n k_1^{-n} \cdots k_p^{-n}}{(b_1)_n \cdots (b_q)_n (\rho)_n l_1^{-n} \cdots l_q^{-n}} \frac{(c)^{(n+\gamma)\alpha-\beta-1}}{n! \Gamma((n + \gamma)\alpha - \beta)} D_{z,q}^\mu z^{(n+\gamma)\alpha-\beta-1+\lambda-1}
\]

Hence the statement (2.1) is proved.

REFERENCES


