ABSTRACT
Rateless codes are referred to as a class of digital fountain codes. Luby Transform codes are first practical erasure-correcting codes which efficaciously improves the performance of systems used for wireless data transmission. This paper investigated the performance of LT codes over additive white Gaussian noise channel. A new model is proposed for wireless broadcast system by concatenating LT codes with Convolutional codes to achieve higher Bit Error Rate (BER). We analyses BER performance of LT coded OFDM systems over AWGN and Rayleigh faded channel with BPSK modulation. Also the comparison is carried out between the LT codes and proposed model. The simulation results in terms of BER shows that proposed model has a coding gain of 4 dB as opposed to conventional LT codes.

KEYWORDS: Fountain codes; Rateless codes; Rayleigh fading Channel; OFDM; AWGN; LT codes; Convolution codes; BER; SNR.

INTRODUCTION
Broadcasting in wireless environment is gaining new heights in the technological domain. Therefore, it is necessary to achieve better data rates and high Quality-of-service (QoS), wide coverage and low operational costs. Various methods have been developed with the aim of checking the pervasiveness of errors in transmitted signals. Numerous error correcting codes like Linear Block Codes, Convolution codes, Reed-Solomon Codes, LDPC codes are developed since as a solution to realize better performance for various communication technologies.

Rateless codes [1] are a class of erasure-correcting that requires no feedback with simple encoding and decoding structures. LT codes [2] are first class of practical fountain codes [3]. Earlier LT codes are only used for erasure channel but researchers have found out that they can also be used for error correcting codes. LT codes are rateless in the sense that from given source of information they can generate limitless encoding symbol. The decoding of LT codes can recover any subset of data from the generated encoding symbols which are slightly larger than the input symbols. LT codes are based on the construction of Low Density parity check matrix (LDGM) [4], LDGM exhibit high error floor [5] and as the LT codes grows the encoding and decoding complexity grows. Shokrollahi addressed this problem in [6], a LT code is preceded by a high-rate code. Examples of the pre-code include low-density parity check (LDPC) codes [7-8], convolutional codes [8]. As a result, Rateless coding has also been investigated for noisy channels such as Additive White Gaussian noise channel [9-12] and fading channel [13] using OFDM [17] systems. It has been analyzed that with zero erasure bound LT codes perform poor in fading channels. To achieve optimal performance many new modified degree distributions have been discussed in [14-15] to maximize throughput, minimize encoding/decoding delay.

As we know LT codes perform better over BEC but exhibit high bit error rate (BER) and error floor over noisy channels. In this paper, we analyzed the performance of LT codes using OFDM systems over AWGN channel with binary phase shift keying[16] (BPSK) modulation. LT codes are developed using the conventional bipartite graph and using Belief propagation decoding. The contribution to this paper is a development of new design methodology of LT codes over AWGN and fading channel by making use of our framework. The proposed framework concatenates conventional LT codes with high fixed rate codes, in our design we uses convolutional codes. Using concatenated codes the BER drops significantly and decreases encoding and decoding complexity.
The rest of paper is organized as follows. In section II, we describe the LT coding Technique. Section III is devoted to system design. In section IV analysis of simulation results discussed. Section V discussed conclusion drawn from the research work carried out as a part of study.

**LT CODING TECHNIQUE**

**Degree Distribution**

**Ideal Soliton Degree Distribution**

The probability distribution on the random degree of encoding symbols \( \rho(d) \) is a critical part of the design to ensure complete recovery of the original data from the minimum number of encoding symbols. Encoding symbols must have a highly variable degree in order to ensure that the recovery process succeeds. Many encoding symbols must have a low degree so that the recovery process can start and continue to keep the total number of addition operations involved in the generation and recovery process small, whereas some encoding symbols have higher degree to ensure that the recovery process continues until all source data are recovered. Luby introduced the Robust Soliton distribution, which is based on the Ideal Soliton distribution, defined as

\[
\rho(1) = \frac{1}{K}, \quad \rho(d) = \frac{1}{d(d-1)} \quad \text{for} \quad d = 2, 3 \ldots K
\]

**Robust Soliton Degree Distribution**

Unfortunately, it performs very poorly in practice. However, a small modification can fix this to obtain the Robust Soliton distribution \( RS(k,c,\delta) \). This has two extra parameters, \( c \) and \( \delta \). The parameter \( \delta \) is the allowable failure probability of the decoder to recover the data for a given number \( k \) of encoding symbols. Let \( S = c \ln \left( \frac{K}{\delta} \right) \) for some suitable constant \( c > 0 \)

\[
T(d) = \frac{S}{K} \ln \frac{S}{d} \quad \text{for} \quad d = 2, 3 \ldots [\frac{K}{S}]
\]

\[
T(d) = 0 \quad \text{for} \quad d > [\frac{K}{S}]
\]

Here, \( 0 < \delta < 1 \) is a (conservative) bound on the probability that the decoding fails to succeed after a certain number of packets are received. \( c > 0 \) is a free parameter, which can be tuned to optimize performance. The robust soliton distribution is

\[
\mu(d) = \rho(d) + T(d) \quad \text{Z} = \sum_{d=1}^{K} \rho(d) + T(d)
\]

The inclusion of \( Z \) creates a properly normalized distribution which sums to one.

The robust soliton distribution defines the distribution \( \mu(d) \) which you will use when implementing your encoder. To sample from \( \mu(d) \), first compute the corresponding cumulative distribution function:

\[
M(d) = \sum_{d'=1}^{d} \mu(d')
\]

Let \( u \) denote a number uniformly distributed between 0 and 1, for example drawn from the pseudo-random generator. We can then construct a sample \( d \) from \( \mu(d) \) by finding the unique bin (degree) for which

\( M(d - 1) < u < M(d) \), where \( M(0) = 0 \).

Hence, for the sake of improving the LT code’s performance in hostile wireless channels, we specifically design the LT code’s degree distribution by expanding its generator matrix with the aid of a unity matrix having a size of \( (K \times K) \), which results in a systematic LT code.

**LT Encoding Process and Decoding Process**

In LT encoding the \( K \) information symbols is divided into source symbols \( s_1, s_2, s_3 \ldots s_K \) to produce \( N \) encoded symbols \( N_1, N_2, N_3 \ldots N_N \). The encoded symbols are produced based on Degree distribution. The degree \( d \) produced should be less than the \( K \) information symbols. Choose uniformly at random \( d \) distinct information symbols to produce
encoded symbols which are modulo 2 sum of different d distinct information symbols. The generator matrix G is constructed based on degree distribution.

LT codes decoding process uses Belief propagation algorithm. The algorithm starts with to check degree 1 of the encoded symbol. The degree-one coded symbol has transferred its content to its one neighboring information symbol, and the corresponding edge is eliminated. The recovered information symbol is added modulo 2 to each connected coded symbol to update its content. Then, the corresponding edges are eliminated. This process continues until all the symbols are recovered. If there is no degree-1 encoded symbol, the decoding process halts. Therefore, a decoding failure rate δ is chosen.

SYSTEM DESIGN
LT codes are referred to as rateless in the sense it generates infinite encoding symbol. An input grey scale image of size 256x256 is given as input to LT encoder. From the K input symbols the encoding symbol can be generated, independently of all other encoding symbols, on average by \(O(\ln(\frac{K}{\delta}))\) symbol operations. Based on degree distribution, LT encoder produces \(N\) encoded symbols using XOR operations of the specific input symbols. LT codes perform better when concatenated with fixed high rate codes. These encoded symbols are given as input to convolutional encoder with rate \(R = \frac{1}{2}\) and are modulated using BPSK modulator. Modulated output is orthogonally frequency division multiplexed, as nowadays OFDM technique become popular because of its high spectral efficiency and robustness against frequency selective fading. The IFFT of the input symbols and cyclic prefixes are added. In our simulation we use 64 point FFT. To analyse the performance of LT codes in fading environment we use Rayleigh fading channel and Gaussian noise added to the signal.

At the receiver side LT decoder can recover \(K\) input symbols from any \(K + O(\sqrt{K \ln(\frac{K}{\delta}))}\) encoding symbols on average \(O(K \ln(\frac{K}{\delta})))\) symbol operations with probability \(1 - \delta\). The reverse operation is performed OFDM is computed FFT and cyclic prefix removes. The signal is demodulated using BPSK demodulator and then it is passed through convolutional decoder using Viterbi algorithm. LT decoder uses belief propagation algorithm to decode the \(K\) input symbols.

**SIMULATION RESULTS**

LT codes based on Belief propagation decoding algorithm is analyzed and compared with conventional LT codes and proposed concatenated codes. Simulation is carried out using MATLAB. The LT encoder with parameter \(c = 0.1\) and \(\delta = 0.5\) is concatenated with convolutional encoder with parameters given in Table 1

<table>
<thead>
<tr>
<th>SNO</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Input Image size</td>
<td>256x256</td>
</tr>
<tr>
<td>2</td>
<td>Number of Bits</td>
<td>524288</td>
</tr>
<tr>
<td>3</td>
<td>Coding</td>
<td>LT codes + convolutional codes</td>
</tr>
<tr>
<td>4</td>
<td>Modulation</td>
<td>BPSK</td>
</tr>
<tr>
<td>5</td>
<td>Code Rate</td>
<td>1/2</td>
</tr>
<tr>
<td>6</td>
<td>OFDM FFT size</td>
<td>64</td>
</tr>
<tr>
<td>7</td>
<td>Noise Channel</td>
<td>AWGN, Rayleigh</td>
</tr>
<tr>
<td>8</td>
<td>SNR</td>
<td>1-20dB</td>
</tr>
<tr>
<td>9</td>
<td>(c)</td>
<td>0.1</td>
</tr>
<tr>
<td>10</td>
<td>(\delta)</td>
<td>0.5</td>
</tr>
</tbody>
</table>

The Fig. 4 depicts the comparison of BER vs SNR curve for LT codes and concatenated LT codes with convolutional codes. It is analyzed there is significant improvement of proposed concatenated codes.

Fig 6 depicts the comparison of BER vs SNR curve for LT codes and concatenated LT codes with convolutional codes. It is analyzed there is significant improvement of proposed concatenated codes.

**CONCLUSION**

In this paper concatenated codes of LT codes and convolutional codes are proposed and their performance is analysed. The simulation results shows that the proposed concatenated codes provide lower BER than conventional LT codes in AWGN and Rayleigh Faded Channel.

Fig 4 Performance comparison of conventional LT codes and concatenated codes in AWGN channel

Fig. 5 comparison of 256x256 Images in AWGN channel at 4 dB where (a) is the input image, (b) conventional LT codes , (c) proposed concatenated codes

Fig 6 Performance comparison of conventional LT codes and concatenated codes in Rayleigh Faded Channel
REFERENCES