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## IJESKI INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY HYPER DOWNHILL INDICES AND THEIR POLYNOMIALS OF CERTAIN CHEMICAL DRUGS

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## ABSTRACT

In this paper, we introduce the first and second hyper downhill indices and their polynomials of a graph. Also we determine these newly defined the first and second hyper downhill indices and their polynomials for some important chemical drugs such as chloroquine, hydroxychloroquins and remdesivir.

**KEYWORDS:** hyper downhill indices, hyper downhill polynomials, chemical structure.

Mathematics Subject Classification: 05C10, 05C69

#### 1. INTRODUCTION

In this paper, G denotes a finite, simple, connected graph, V(G) and E(G) denote the vertex set and edge set of G.  $d_{-}(u)$ 

The degree  $d_G(u)$  of a vertex u is the number of vertices adjacent to u. A u-v path P in G is a sequence of vertices in G, starting with u and ending at V, such that consecutive vertices in P are adjacent, and no vertex is repeated.

A path  $\pi = v_1, v_2, \dots, v_{k+1}$  in G is a downhill path if for every i,  $1 \le i \le k$ ,  $d_G(v_i) \ge d_G(v_{i+1})$ .

A vertex v is downhill dominates a vertex u if there exists a downhill path originated from u to v. The downhill neighborhood of a vertex v is denoted by  $N_{dn}(v)$  and defined as:  $N_{dn}(v) = \{u: v \text{ downhill dominates } u\}$ . The downhill degree  $d_{dn}(v)$  of a vertex v is the number of downhill neighbors of v [1].

The first and second downhill Zagreb indices were introduced by Al-Ahmadi et al. in [1] and defined as

$$DWM_{1}(G) = \sum_{uv \in E(G)} (d_{dn}(u))^{2}.$$
$$DWM_{2}(G) = \sum_{uv \in E(G)} d_{dn}(u) d_{dn}(v)$$

Recently, some downhill indices were studied in [2, 3, 4, 5].

A new version of the first downhill index is defined as

$$DWM_{1}^{*}(G) = \sum_{uv \in E(G)} [d_{dn}(u) + d_{dn}(v)].$$

We now define the first and second hyper downhill indices as

$$HDW_{1}(G) = \sum_{uv \in E(G)} \left[ d_{dn}(u) + d_{dn}(v) \right]^{2}$$

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$$HDW_{2}(G) = \sum_{uv \in E(G)} \left[ d_{dn}(u) d_{dn}(v) \right]^{2}.$$

Considering the first and second hyper downhill indices, we propose the first and second hyper downhill polynomials of G, defined respectively as

$$DWM_{1}^{*}(G, x) = \sum_{uv \in E(G)} x^{d_{dn}(u) + d_{dn}(v)}$$
$$HDW_{1}(G, x) = \sum_{uv \in E(G)} x^{\left[d_{dn}(u) + d_{dn}(v)\right]^{2}}$$
$$HDW_{2}(G, x) = \sum_{uv \in E(G)} x^{\left[d_{dn}(u)d_{dn}(v)\right]^{2}}.$$

Recently, some topological indices were studied in [6-11].

In this paper, we compute the first and second hyper downhill indices and their polynomials of chloroquine, hydroxychloroquine and remdesivir.

#### 2. RESULTS AND DISCUSSION: CHLOROQUINE

Chloroquine is an antiviral compound (drug) which was discovered in 1934 by H.Andersag. This drug is medication primarily used to prevent and treat malaria.

Let G be the chemical structure of chloroqine. This structure has 21 vertices and 23 edges.

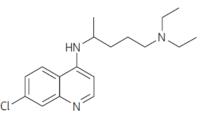


Figure 1. Chemical structure of chloroquine

From Figure 1, we obtain that

 $\{(d_{dn}(u), d_{dn}(v)) \mid uv \square E(G)\}$  has 13 edge set partitions.

$d_{dn}(u), d_{dn}(v)$ uv E(G) Number of edges	(9,9) 2 (2,5) 1 (0,5) 2	(2,9) 2 (1,4) 1 (0,4) 2	(1,9) 1 (2,2) 4 (0,1) 2	(2,7) 1 (1,1) 1	(1,7) 2 (0,9) 2
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We calculate the first downhill index and its polynomial of chloroquine as follows.

Theorem 1. Let G be the chemical structure of chloroquine. Then

(i)  $DWM_1^*(G) = 161$ 

(ii)  $DWM_1^*(G, x) = 2x^{18} + 2x^{11} + 1x^{10} + 3x^9 + 2x^8 + 1x^7 + 3x^5 + 6x^4 + 1x^2 + 2x^1$ 

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Proof: By using the definitions and edge partition of G, we deduce

$$DWM_{1}^{*}(G) = \sum_{uv \in E(G)} \left[ d_{dn}(u) + d_{dn}(v) \right]$$
  
= (9+9)2+(2+9)2+(1+9)1+(2+7)1+(1+7)2+(2+5)1+(1+4)1  
+(2+2)4+(1+1)1+(0+9)2+(0+5)2+(0+4)2+(0+1)2

By simplifying the above equation, we get desired result.

(ii)

(i)

$$DWM_1^*(G, x) = \sum_{uv \in E(G)} x^{d_{dn}(u) + d_{dn}(v)}$$
  
=  $2x^{9+9} + 2x^{2+9} + 1x^{1+9} + 1x^{2+7} + 2x^{1+7} + 1x^{2+5} + 1x^{1+4}$   
 $+ 4x^{2+2} + 1x^{1+1} + 2x^{0+9} + 2x^{0+5} + 2x^{0+4} + 2x^{0+1}$ 

By simplifying the above equation, we get desired result.

We calculate the first hyper downhill index and its polynomial of chloroquine as follows.

Theorem 2. Let G be the chemical structure of chloroquine. Then

(i) 
$$HDW_1(G) = 1587$$
  
(ii)  $HDW_1(G, x) = 2x^{324} + 2x^{121} + 1x^{100} + 3x^{81} + 2x^{64} + 1x^{49} + 3x^{25} + 6x^{16} + 1x^4 + 2x^1$ 

Proof: By using the definitions and edge partition of G, we deduce

$$HDW_{1}(G) = \sum_{uv \in E(G)} \left[ d_{dn}(u) + d_{dn}(v) \right]^{2}$$
  
=  $(9+9)^{2} 2 + (2+9)^{2} 2 + (1+9)^{2} 1 + (2+7)^{2} 1 + (1+7)^{2} 2 + (2+5)^{2} 1 + (1+4)^{2} 1 + (2+2)^{2} 4 + (1+1)^{2} 1 + (0+9)^{2} 2 + (0+5)^{2} 2 + (0+4)^{2} 2 + (0+1)^{2} 2$ 

By simplifying the above equation, we get desired result.

(ii) 
$$HDW_{1}(G, x) = \sum_{uv \in E(G)} x^{\left[d_{dn}(u) + d_{dn}(v)\right]^{2}}$$

$$= 2x^{(9+9)^2} + 2x^{(2+9)^2} + 1x^{(1+9)^2} + 1x^{(2+7)^2} + 2x^{(1+7)^2} + 1x^{(2+5)^2} + 1x^{(1+4)^2} + 4x^{(2+2)^2} + 1x^{(1+1)^2} + 2x^{(0+9)^2} + 2x^{(0+5)^2} + 2x^{(0+4)^2} + 2x^{(0+1)^2}$$

By simplifying the above equation, we get desired result.

We calculate the second hyper downhill index and its polynomial of chloroquine as follows.

Theorem 3. Let G be the chemical structure of chloroquine. Then

(i) 
$$HDW_2(G) = 14326$$
  
(ii)  $HDW_2(G, x) = 2x^{6561} + 2x^{324} + 1x^{81} + 1x^{196} + 2x^{49} + 1x^{100} + 5x^{16} + 1x^1 + 8$ 

Proof: By using the definitions and edge partition of G, we deduce

(i)  

$$HDW_{2}(G) = \sum_{uv \in E(G)} \left[ d_{dn}(u) d_{dn}(v) \right]^{2}$$

$$= (9 \times 9)^{2} 2 + (2 \times 9)^{2} 2 + (1 \times 9)^{2} 1 + (2 \times 7)^{2} 1 + (1 \times 7)^{2} 2 + (2 \times 5)^{2} 1 + (1 \times 4)^{2} 1 + (2 \times 2)^{2} 4 + (1 \times 1)^{2} 1 + (0 \times 9)^{2} 2 + (0 \times 5)^{2} 2 + (0 \times 4)^{2} 2 + (0 \times 1)^{2} 2$$

By simplifying the above equation, we get desired result.

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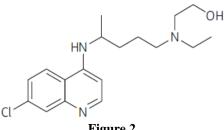
 $HDW_2$ 

$$(G, x) = \sum_{uv \in E(G)} x^{\left[d_{dn}(u)d_{dn}(v)\right]^{2}}$$
  
=  $2x^{(9\times9)^{2}} + 2x^{(2\times9)^{2}} + 1x^{(1\times9)^{2}} + 1x^{(2\times7)^{2}} + 2x^{(1\times7)^{2}} + 1x^{(2\times5)^{2}} + 1x^{(1\times4)^{2}}$   
+ $4x^{(2\times2)^{2}} + 1x^{(1\times1)^{2}} + 2x^{(0\times9)^{2}} + 2x^{(0\times5)^{2}} + 2x^{(0\times4)^{2}} + 2x^{(0\times1)^{2}}$ 

By simplifying the above equation, we get desired result.

### 3. RESULTS AND DISCUSSION: HYDROXYCHLOROQUINE

Let H be the molecular structure of hydroxychloroquine. Clearly H has 22 vertices and 24 edges, see Figure 2.





From Figure 2, we obtain that

 $\{(d_{dn}(u), d_{dn}(v) \mid uv \square E(H)\}$  has 14 edge set partitions

$d_{dn}(u), d_{dn}(v) $ \ uv E(H) Number of edges	(9,9) 2 (2,5) 1 (0,5) 2	(2,9) 2 (1,4) 1 (0,4) 2	(1,9)  1  (2,2)  5  (0,2)  1	(2,8) 2 (1,1) 1 (0,1) 1	(1,8) 1 (0,9) 2	
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We calculate the first downhill index and its polynomial of hydroxychloroquine as follows.

Theorem 4. Let H be the chemical structure of hydroxychloroquine. Then

(i)  $DWM_1^*(H) = 170$ 

(ii) 
$$DWM_1^*(H, x) = 2x^{18} + 2x^{11} + 3x^{10} + 3x^9 + 1x^7 + 3x^5 + 7x^4 + 2x^2 + 1x^1$$
.

Proof: By using the definitions and edge partition of H, we deduce

(i)  

$$DWM_{1}^{*}(H) = \sum_{uv \in E(H)} \left[ d_{dn}(u) + d_{dn}(v) \right]$$

$$= (9+9)2 + (2+9)2 + (1+9)1 + (2+8)2 + (1+8)1 + (2+5)1 + (1+4)1 + (2+2)5 + (1+1)1 + (0+9)2 + (0+5)2 + (0+4)2 + (0+2)1 + (0+1)1 + (0+1)1 + (0+1)2 + (0+2)1 + (0+1)1 + (0+1)2 + (0+2)1 + (0+1)1 + (0+1)2 + (0+2)1 + (0+1)1 + (0+1)2 + (0+2)1 + (0+1)1 + (0+1)2 + (0+1)$$

By simplifying the above equation, we get desired result.

$$DWM_1^*(H, x) = \sum_{uv \in E(H)} x^{d_{dn}(u) + d_{dn}(v)}$$

(ii)

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$$= 2x^{9+9} + 2x^{2+9} + 1x^{1+9} + 2x^{2+8} + 1x^{1+8} + 1x^{2+5} + 1x^{1+4}$$
  
+5x<sup>2+2</sup> + 1x<sup>1+1</sup> + 2x<sup>0+9</sup> + 2x<sup>0+5</sup> + 2x<sup>0+4</sup> + 1x<sup>0+2</sup> + 1x<sup>0+1</sup>

By simplifying the above equation, we get desired result.

We calculate the first hyper downhill index and its polynomial of hydroxychloroquine as follows.

Theorem 5. Let H be the chemical structure of hydroxychloroquine. Then

(i) 
$$HDW_1(H) = 1678$$

 $HDW_1(H, x) = 2x^{324} + 2x^{121} + 3x^{100} + 3x^{81} + 1x^{49} + 3x^{25} + 7x^{16} + 2x^4 + 1x^1.$ (ii)

Proof: By using the definitions and edge partition of H, we deduce

$$HDW_{1}(H) = \sum_{uv \in E(H)} \left[ d_{dn}(u) + d_{dn}(v) \right]^{2}$$
  
=  $(9+9)^{2} 2 + (2+9)^{2} 2 + (1+9)^{2} 1 + (2+8)^{2} 2 + (1+8)^{2} 1 + (2+5)^{2} 1 + (1+4)^{2} 1 + (2+2)^{2} 5 + (1+1)^{2} 1 + (0+9)^{2} 2 + (0+5)^{2} 2 + (0+4)^{2} 2 + (0+2)^{2} 1 + (0+1)^{2} 1$ 

By simplifying the above equation, we get desired result.

(ii)

 $HDW_1$ 

$$(H, x) = \sum_{uv \in E(H)} x^{\left[d_{dn}(u) + d_{dn}(v)\right]^2}$$
  
=  $2x^{(9+9)^2} + 2x^{(2+9)^2} + 1x^{(1+9)^2} + 2x^{(2+8)^2} + 1x^{(1+8)^2} + 1x^{(2+5)^2} + 1x^{(1+4)^2}$   
+ $5x^{(2+2)^2} + 1x^{(1+1)^2} + 2x^{(0+9)^2} + 2x^{(0+5)^2} + 2x^{(0+4)^2} + 1x^{(0+2)^2} + 1x^{(0+1)^2}$ 

By simplifying the above equation, we get desired result.

We calculate the second hyper downhill index and its polynomial of hydroxychloroquine as follows.

Theorem 6. Let H be the chemical structure of hydroxychloroquine. Then

(i) 
$$HDW_2(H) = 14624$$
  
(ii)  $HDW_2(H, x) = 2x^{6561} + 2x^{324} + 1x^{81} + 2x^{256} + 1x^{64} + 1x^{100} + 6x^{16} + 1x^{1} + 8$ 

Proof: By using the definitions and edge partition of H, we deduce

(i)  

$$HDW_{2}(H) = \sum_{uv \in E(H)} \left[ d_{dn}(u) d_{dn}(v) \right]^{2}$$

$$= (9 \times 9)^{2} 2 + (2 \times 9)^{2} 2 + (1 \times 9)^{2} 1 + (2 \times 8)^{2} 2 + (1 \times 8)^{2} 1 + (2 \times 5)^{2} 1 + (1 \times 4)^{2} 1 + (2 \times 2)^{2} 5 + (1 \times 1)^{2} 1 + (0 \times 9)^{2} 2 + (0 \times 5)^{2} 2 + (0 \times 4)^{2} 2 + (0 \times 2)^{2} 1 + (0 \times 1)^{2} 1$$

By simplifying the above equation, we get desired result.  $H, x) = \sum_{uv \in E(H)} x^{\left[d_{dn}(u)d_{dn}(v)\right]^2}$ 

$$= 2x^{(9\times9)^{2}} + 2x^{(2\times9)^{2}} + 1x^{(1\times9)^{2}} + 2x^{(2\times8)^{2}} + 1x^{(1\times8)^{2}} + 1x^{(2\times5)^{2}} + 1x^{(1\times4)^{2}} + 5x^{(2\times2)^{2}} + 1x^{(1\times1)^{2}} + 2x^{(0\times9)^{2}} + 2x^{(0\times5)^{2}} + 2x^{(0\times4)^{2}} + 1x^{(0\times2)^{2}} + 1x^{(0\times1)^{2}}$$

By simplifying the above equation, we get desired result.

### 4. RESULTS AND DISCUSSION: REMDESIVIR

Let R be the molecular structure of remdesivir. Clearly R has 41 vertices and 44 edges, see Figure 3.

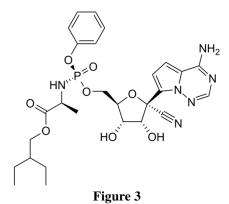
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From Figure 3, we find that

 $\{(d_{dn}(u), d_{dn}(v)) \setminus uv \square E(R)\}$  has 20 bond set partitions.

Table 3, Edge set partitions of remdesivir

$d_{dn}(u), d_{dn}(v)$ \uv $\in E(R)$ Number of edges	(9,19) 1 (4,6) 2 (0,7) 3	(7,19) 1 (1,6) 4 (0,6) 4	(9,9) 3 (1,5) 1 (0,5) 3	(2,9) 2 (4,4) 4 (0,1) 2	(1,9) 2 (2,2) 2	(7,7) 2 (1,1) 3	(1,7) 1 (0,19) 2	(6,6) 1 (0,9) 1	
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We calculate the first downhill index and its polynomial of remdesivir as follows.

Theorem 7. Let R be the chemical structure of remdesivir. Then

(i)  $DWM_1^*(R) = 407$ 

(ii) 
$$DWM_{1}^{*}(R,x) = 1x^{28} + 1x^{26} + 3x^{18} + 2x^{11} + 4x^{10} + 2x^{14} + 5x^{8} + 1x^{12} + 7x^{7} + 5x^{6} + 2x^{4} + 3x^{2} + 2x^{19} + 1x^{9} + 3x^{5} + 2x^{1}$$

Proof: By using the definitions and edge partition of R, we deduce  $DWM_{1}^{*}(R) = \sum_{uv \in E(R)} \left[ d_{dn}(u) + d_{dn}(v) \right]$ 

(i)

$$= (9+19)1 + (7+19)1 + (9+9)3 + (2+9)2 + (1+9)2 + (7+7)2 + (1+7)1 + (6+6)1 + (4+6)2 + (1+6)4 + (1+5)1 + (4+4)4 + (2+2)2 + (1+1)3 + (0+19)2 + (0+9)1 + (0+7)3 + (0+6)4 + (0+5)3 + (0+1)2$$

By simplifying the above equation, we get desired result.

(ii) 
$$DWM_1^*(R,x) = \sum_{uv \in E(R)} x^{d_{dn}(u) + d_{dn}(v)}$$
$$= 1x^{9+19} + 1x^{7+19} + 2x^{9+9} + 1x^{7+19} + 2x^{9+9} + 1x^{7+19} + 2x^{9+9} + 1x^{7+19} + 2x^{9+9} + 1x^{7+19} + 1x^{7} + 1x^{7$$

$$=1x^{9+19} + 1x^{7+19} + 3x^{9+9} + 2x^{2+9} + 2x^{1+9} + 2x^{7+7} + 1x^{1+7} + 1x^{6+6} + 2x^{4+6} + 4x^{1+6} + 1x^{1+5} + 4x^{4+4} + 2x^{2+2} + 3x^{1+1} + 2x^{0+19} + 1x^{0+9} + 3x^{0+7} + 4x^{0+6} + 3x^{0+5} + 2x^{0+1}$$

By simplifying the above equation, we get desired result.

We calculate the first hyper downhill index and its polynomial of remdesivir as follows.

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Theorem 8. Let R be the chemical structure of remdesivir. Then

(i)  $HDW_1(R) = 5377$ (ii)  $HDW_1(R, x) = 1x^{784} + 1x^{676} + 3x^{324} + 2x^{121} + 4x^{100} + 2x^{196} + 5x^{64} + 1x^{144} + 7x^{49} + 5x^{36} + 2x^{16} + 3x^4 + 2x^{361} + 1x^{81} + 3x^{25} + 2x^{1}$ 

Proof: By using the definitions and edge partition of R, we deduce

(i)  
$$HDW_{1}(R) = \sum_{uv \in E(R)} \left[ d_{dn}(u) + d_{dn}(v) \right]^{2}$$
$$(0 + 10)^{2} 1 + (7 + 10)^{2} 1 + (0 + 0)^{2}$$

$$= (9+19)^{2} 1 + (7+19)^{2} 1 + (9+9)^{2} 3 + (2+9)^{2} 2 + (1+9)^{2} 2 + (7+7)^{2} 2 + (1+7)^{2} 1 + (6+6)^{2} 1 + (4+6)^{2} 2 + (1+6)^{2} 4 + (1+5)^{2} 1 + (4+4)^{2} 4 + (2+2)^{2} 2 + (1+1)^{2} 3 + (0+19)^{2} 2 + (0+9)^{2} 1 + (0+7)^{2} 3 + (0+6)^{2} 4 + (0+5)^{2} 3 + (0+1)^{2} 2$$

By simplifying the above equation, we get desired result.

(ii) 
$$HDW_{1}(R,x) = \sum_{uv \in E(R)} x^{\left[d_{dn}(u) + d_{dn}(v)\right]^{2}}$$

$$=1x^{(9+19)^{2}} + 1x^{(7+19)^{2}} + 3x^{(9+9)^{2}} + 2x^{(2+9)^{2}} + 2x^{(1+9)^{2}} + 2x^{(7+7)^{2}} + 1x^{(1+7)^{2}} + 1x^{(6+6)^{2}} + 2x^{(4+6)^{2}} + 4x^{(1+6)^{2}} + 4$$

 $+1x^{(1+5)^{2}} + 4x^{(4+4)^{2}} + 2x^{(2+2)^{2}} + 3x^{(1+1)^{2}} + 2x^{(0+19)^{2}} + 1x^{(0+9)^{2}} + 3x^{(0+7)^{2}} + 4x^{(0+6)^{2}} + 3x^{(0+5)^{2}} + 2x^{(0+1)^{2}}$ By simplifying the above equation, we get desired result.

We calculate the second hyper downhill index and its polynomial of remdesivir as follows.

Theorem 9. Let R be the chemical structure of remdesivir. Then

(i) 
$$HDW_2(R) = 75950$$
  
(ii)  $HDW_2(R, x) = 1x^{29241} + 1x^{17689} + 3x^{6561} + 2x^{324} + 2x^{81} + 2x^{2401} + 1x^{49} + 1x^{1296} + 2x^{576} + 4x^{36} + 1x^{25} + 4x^{256} + 2x^{16} + 3x^{1} + 15x^{0}$ 

Proof: By using the definitions and edge partition of R, we deduce

(i)  

$$HDW_{2}(R) = \sum_{uv \in E(R)} \left[ d_{dn}(u) d_{dn}(v) \right]^{2}$$

$$= (9 \times 19)^{2} 1 + (7 \times 19)^{2} 1 + (9 \times 9)^{2} 3 + (2 \times 9)^{2} 2 + (1 \times 9)^{2} 2 + (7 \times 7)^{2} 2 + (1 \times 7)^{2} 1 + (6 \times 6)^{2} 1 + (4 \times 6)^{2} 2 + (1 \times 6)^{2} 4 + (1 \times 5)^{2} 1 + (4 \times 4)^{2} 4 + (2 \times 2)^{2} 2 + (1 \times 1)^{2} 3 + (0 \times 19)^{2} 2 + (0 \times 9)^{2} 1 + (0 \times 7)^{2} 3 + (0 \times 6)^{2} 4 + (0 \times 5)^{2} 3 + (0 \times 1)^{2} 2$$

By simplifying the above equation, we get desired result.

$$HDW_{2}(R,x) = \sum_{uv \in E(R)} x^{\left[d_{dn}(u)d_{dn}(v)\right]^{2}}$$

(ii)

$$= 1x^{(9\times19)^2} + 1x^{(7\times19)^2} + 3x^{(9\times9)^2} + 2x^{(2\times9)^2} + 2x^{(1\times9)^2} + 2x^{(7\times7)^2} + 1x^{(1\times7)^2} + 1x^{(6\times6)^2} + 2x^{(4\times6)^2} + 4x^{(1\times6)^2} + 1x^{(1\times5)^2} + 4x^{(4\times4)^2} + 2x^{(2\times2)^2} + 3x^{(1\times1)^2} + 2x^{(0\times19)^2} + 1x^{(0\times9)^2} + 3x^{(0\times7)^2} + 4x^{(0\times6)^2} + 3x^{(0\times5)^2} + 2x^{(0\times1)^2}$$

By simplifying the above equation, we get desired result.

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## 5. CONCLUSION

In this paper, we have introduced the first and second hyper downhill indices and their corresponding polynomials of a graph. Also these newly defined the first and second hyper downhill indices and their polynomials for some important chemical drugs such as chloroquine, hydroxychloroquine and remdesivir are computed.

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