International Journal of Engineering Sciences &Research Technology

(A Peer Reviewed Online Journal)

Impact Factor: 5.164

IJESRT

RESEARCHERID
THOMSON REUTERS

[KULLI , 13(6): June, 2024]
ICTM Value: 3.00

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

IJESRT
INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

EULER SOMBOR BANHATTI INDICES

V.R.Kulli
Department of Mathematics Gulbarga University, Gulbarga 585106, India
e-mail: vrkulli@gmail.com

DOI:10.57030/ijesrt.13.6.2.2024

ABSTRACT

In this study, we introduce the reduced Euler SomborBanhatti index, neighborhood Euler SomborBanhatti index, KV Euler SomborBanhatti index, delta Euler SomborBanhatti index, Revan Euler SomborBanhatti index, leap Euler SomborBanhatti index, status Euler SomborBanhatti index, ve-degree Euler SomborBanhatti index, Kulli-Basava Euler SomborBanhatti index, domination Euler SomborBanhatti index, temperature Euler SomborBanhatti index, Euler Sombor E-Banhatti index and their corresponding exponentials of a graph. Furthermore, we compute these indices and their corresponding exponentials for some graphs.

KEYWORDS: Euler Sombor index, modified Euler Sombor index, graph.

1. INTRODUCTION

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree d_u of a vertex u is the number of vertices adjacent to u. Let $\Delta(G)$ ($\delta(G)$) denote the maximum (minimum) degree among the vertices of G. We refer [1] for undefined notations and terminologies.

Graph indices [2] have their applications in various disciplines of Science and Technology.

2. EULER SOMBOR INDEX

The Euler Sombor index [3] or Nirmala alpha Gourava index [4] of a graph G is defined as

$$EU(G)=\sum_{uv\in E(G)}\sqrt{d_u^{\,2}+d_v^{\,2}+d_ud_v}.$$

Considering the Euler Sombor index, the Euler Sombor exponential [4] of a graph G is defined as

$$EU(G,x)=\sum_{uv\in E(G)}x^{\sqrt{d_u^{\,2}+d_v^{\,2}+d_ud_v}}.$$

The modified Euler Sombor index [4] of a graph G is defined as

$${}^{m}EU(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_u^{\,2}+d_v^{\,2}+d_ud_v}}.$$

Considering the modified Euler Sombor index, the modified Euler Sombor exponential [4] of a graph G is defined as

$${}^{m}EU(G,x)=\sum_{uv\in E(G)}x^{\frac{1}{\sqrt{d_u^{\,2}+d_v^{\,2}+d_ud_v}}}.$$

Recently, some Sombor indices were studied in [5-22].

RESULTS FOR $HC_5C_7[p,q]$ NANOTUBES

In this section, we focus on the family of nanotubes, denoted by $HC_5C_7[p,q]$, in which p is the number of heptagons in the first row and q rows of pentagons repeated alternately. Let G be the graph of a nanotube $HC_5C_7[p,q]$.

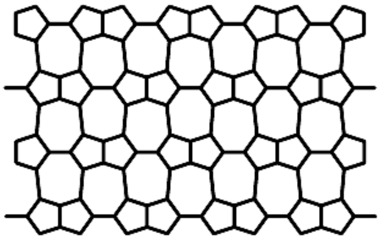

Figure 1. The 2-*D* lattice of nanotubeHC_5C_7 [8, 4]

The 2-D lattice of nanotube $HC_5C_7[p, q]$ is shown in Figure 1.By calculation, we obtain that G has $4pq$ vertices and $6pq - p$ edges. The graph G has two types of edges based on the degree of end vertices of each edge as given in Table 1.

$d_u, d_u / uv \in E(G)$	Number of edges
(2, 3)	$4p$
(3, 3)	$6pq - 5p$

Table 1. Edge partition of $HC_5C_7[p, q]$

Theorem 2.1. Let$G$ be the graph of a nanotube $HC_5C_7[p, q]$.Then

$$EU(G) = 6\sqrt{27}\,pq + \left(4\sqrt{19} - 5\sqrt{27}\right)p.$$

Proof: From definition and by using Table 1, we deduce

$$EU(G) = \sum_{uv \in E(G)} \sqrt{d_u^{\,2} + d_v^{\,2} + d_u d_v}$$

$$= 4p\sqrt{2^2 + 3^2 + 2\times 3} + (6pq - 5p)\sqrt{3^2 + 3^2 + 3\times 3}$$

$$= 6\sqrt{27}\,pq + \left(4\sqrt{19} - 5\sqrt{27}\right)p.$$

Theorem 2.2. Let$G$ be the graph of a nanotube $HC_5C_7[p, q]$.Then

$$EU(G, x) = 4px^{\sqrt{19}} + (6pq - 5p)x^{\sqrt{27}}.$$

Proof: From definition and by using Table 1, we obtain

$$EU(G, x) = \sum_{uv \in E(G)} x^{\sqrt{d_u^{\,2} + d_v^{\,2} + d_u d_v}}$$

$$= 4px^{\sqrt{2^2 + 3^2 + 2\times 3}} + (6pq - 5p)x^{\sqrt{3^2 + 3^2 + 3\times 3}}$$

$$= 4px^{\sqrt{19}} + (6pq - 5p)x^{\sqrt{27}}.$$

Theorem 2.3. Let$G$ be the graph of a nanotube $HC_5C_7[p, q]$.Then

$$^{m}EU(G) = \frac{6pq}{\sqrt{27}} + \frac{4p}{\sqrt{19}} - \frac{5p}{\sqrt{27}}.$$

Proof: From definition and by using Table 1, we deduce

$$^{m}EU(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_u^{\,2} + d_v^{\,2} + d_u d_v}}$$

$$= \frac{4p}{\sqrt{2^2 + 3^2 + 2\times 3}} + \frac{6pq - 5p}{\sqrt{3^2 + 3^2 + 3\times 3}}$$

$$= \frac{6pq}{\sqrt{27}} + \frac{4p}{\sqrt{19}} - \frac{5p}{\sqrt{27}}.$$

Theorem 2.4. Let G be the graph of a nanotube $HC_5C_7[p, q]$. Then

$$^{m}EU(G,x) = 4px^{\frac{1}{\sqrt{19}}} + (6pq - 5p)x^{\frac{1}{\sqrt{27}}}.$$

Proof: From definition and by using Table 1, we get

$$^{m}EU(G,x) = \sum_{uv\in E(G)} x^{\frac{1}{\sqrt{d_u^2+d_v^2+d_ud_v}}}$$

$$= 4px^{\frac{1}{\sqrt{2^2+3^2+2\times3}}} + (6pq-5p)x^{\frac{1}{\sqrt{3^2+3^2+3\times3}}}$$

$$= 4px^{\frac{1}{\sqrt{19}}} + (6pq-5p)x^{\frac{1}{\sqrt{27}}}.$$

3. REDUCED EULER SOMBOR BANHATTI INDEX

We introduce a new graph index defined as

$$REEU(G) = \sum_{uv\in E(G)} \sqrt{(d_u-1)^2+(d_u-1)^2+(d_u-1)(d_v-1)}$$

which we propose to be named as reduced Euler SomborBanhatti index.

Considering the reduced Euler SomborBanhatti index, we introduce the reduced Euler SomborBanhatti exponential of a graph G and defined it as

$$REEU(G,x) = \sum_{uv\in E(G)} x^{\sqrt{(d_u-1)^2+(d_u-1)^2+(d_u-1)(d_v-1)}}.$$

We define the modified reduced Euler SomborBanhatti index of a graph G as

$$^{m}REEU(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{(d_u-1)^2+(d_u-1)^2+(d_u-1)(d_v-1)}}.$$

Considering the modified reduced Euler SomborBanhatti index, we introduce the modified reduced Euler SomborBanhatti exponential of a graph G and defined it as

$$^{m}REEU(G,x) = \sum_{uv\in E(G)} x^{\frac{1}{\sqrt{(d_u-1)^2+(d_u-1)^2+(d_u-1)(d_v-1)}}}.$$

RESULTS FOR $HC_5C_7[p,q]$ NANOTUBES

Theorem 3.1. Let G be the graph of a nanotube $HC_5C_7[p, q]$. Then

$$REEU(G) = 6\sqrt{12}pq + (4\sqrt{7} - 5\sqrt{12})p.$$

Proof: From definition and by using Table 1, we obtain

$$REEU(G) = \sum_{uv\in E(G)} \sqrt{(d_u-1)^2+(d_u-1)^2+(d_u-1)(d_v-1)}$$

$$= 4p\sqrt{1^2+2^2+1\times2} + (6pq-5p)\sqrt{2^2+2^2+2\times2}$$

$$= 6\sqrt{12}pq + (4\sqrt{7} - 5\sqrt{12})p.$$

Theorem 3.2. Let G be the graph of a nanotube $HC_5C_7[p, q]$. Then

$$REEU(G,x) = 4px^{\sqrt{7}} + (6pq-5p)x^{\sqrt{12}}.$$

Proof: From definition and by using Table 1, we deduce

$$REEU(G,x) = \sum_{uv\in E(G)} x^{\sqrt{(d_u-1)^2+(d_u-1)^2+(d_u-1)(d_v-1)}}$$

$$= 4px^{\sqrt{1^2+2^2+1\times2}} + (6pq-5p)x^{\sqrt{2^2+2^2+2\times2}}$$

$$= 4px^{\sqrt{7}} + (6pq - 5p)x^{\sqrt{12}}.$$

Theorem 3.3. Let G be the graph of a nanotube $HC_5C_7[p, q]$. Then

$$^{m}REEU(G) = \frac{6pq}{\sqrt{12}} + \frac{4p}{\sqrt{7}} - \frac{5p}{\sqrt{12}}.$$

Proof: From definition and by using Table 1, we get

$$^{m}REEU(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{(d_u - 1)^2 + (d_u - 1)^2 + (d_u - 1)(d_v - 1)}}$$

$$= \frac{4p}{\sqrt{1^2 + 2^2 + 1 \times 2}} + \frac{6pq - 5p}{\sqrt{2^2 + 2^2 + 2 \times 2}}$$

$$= \frac{6pq}{\sqrt{12}} + \frac{4p}{\sqrt{7}} - \frac{5p}{\sqrt{12}}.$$

Theorem 3.4. Let G be the graph of a nanotube $HC_5C_7[p, q]$. Then

$$^{m}REEU(G, x) = 4px^{\frac{1}{\sqrt{7}}} + (6pq - 5p)x^{\frac{1}{\sqrt{12}}}.$$

Proof: From definition and by using Table 1, we deduce

$$^{m}REEU(G, x) = \sum_{uv \in E(G)} x^{\frac{1}{\sqrt{(d_u - 1)^2 + (d_u - 1)^2 + (d_u - 1)(d_v - 1)}}}$$

$$= 4px^{\frac{1}{\sqrt{1^2 + 2^2 + 1 \times 2}}} + (6pq - 5p)x^{\frac{1}{\sqrt{2^2 + 2^2 + 2 \times 2}}}$$

$$= 4px^{\frac{1}{\sqrt{7}}} + (6pq - 5p)x^{\frac{1}{\sqrt{12}}}.$$

4. NEIGHTBORHOOD EULER SOMBOR BANHATTI INDEX

Let S_u denote the sum of the degrees of all vertices adjacent to a vertex u.
We put forward a new index defined as

$$NEU(G) = \sum_{uv \in E(G)} \sqrt{S_u^2 + S_v^2 + S_u S_v}$$

which we propose to be named as neighborhood Euler SomborBanhatti index.
Considering the neighborhood Euler SomborBanhatti index, we introduce the neighborhood Euler SomborBanhatti exponential of a graph G and defined it as

$$NEU(G, x) = \sum_{uv \in E(G)} x^{\sqrt{S_u^2 + S_v^2 + S_u S_v}}.$$

We put forward the modified neighborhood Euler SomborBanhatti index of a graph G and defined it as

$$^{m}NEU(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{S_u^2 + S_v^2 + S_u S_v}}.$$

Considering the modified neighborhood Euler SomborBanhatti index, we introduce the modified neighborhood Euler SomborBanhatti exponential of a graph G and defined it as

$$^{m}NEU(G, x) = \sum_{uv \in E(G)} x^{\frac{1}{\sqrt{S_u^2 + S_v^2 + S_u S_v}}}.$$

Recently, some neighborhood indices were studied in [23-32].

RESULTS FOR NANOCONES $C_n[k]$

We consider nanocones $C_n[k]$. The molecular structure of $C_4[2]$ is shown in Figure 2.

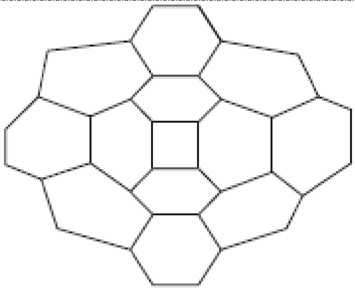

Figure 2. The molecular structure of $C_4[2]$

Let G be the molecular structure of $C_n[k]$. By calculation, G has $n(k+1)^2$ vertices and $\frac{n}{2}(k+1)(3k+2)$ edges. Also by calculation, we obtain that G has five types of edges based on $S_G(u)$ and $S_G(v)$ the degrees of end vertices of each edge as given in Table 2.

$S_G(u), S_G(v) \setminus uv \in E(G)$	Number of edges
(5, 5)	n
(5, 7)	$2n$
(6, 7)	$2(k-1)n$
(7, 9)	nk
(9, 9)	$\frac{nk}{2}(3k-1)$

Table 2. Edge partition of $C_n[k]$ based on $S_G(u)$, $S_G(v)$

Theorem 4.1. Let G be the graph of a nanocone $C_n[k]$. Then

$$NEU(G) == n\sqrt{75} + 2n\sqrt{109} + +2(k-1)n\sqrt{127} + nk\sqrt{193} + \frac{nk}{2}(3k-1)\sqrt{243}.$$

Proof: From definition and by using Table 2, we deduce

$$NEU(G) = \sum_{uv\in E(G)} \sqrt{S_u^2 + S_v^2 + S_u S_v}$$

$$= n\sqrt{5^2+5^2+5\times5} + 2n\sqrt{5^2+7^2+5\times7} + 2(k-1)n\sqrt{6^2+7^2+6\times7}$$

$$+nk\sqrt{7^2+9^2+7\times9} + \frac{nk}{2}(3k-1)\sqrt{9^2+9^2+9\times9}$$

$$= n\sqrt{75} + 2n\sqrt{109} + +2(k-1)n\sqrt{127} + nk\sqrt{193} + \frac{nk}{2}(3k-1)\sqrt{243}.$$

Theorem 4.2. Let G be the graph of a nanocone $C_n[k]$. Then

$$NEU(G,x) = nx^{\sqrt{75}} + 2nx^{\sqrt{109}} + 2(k-1)nx^{\sqrt{127}} + nkx^{\sqrt{193}} + \frac{nk}{2}(3k-1)x^{\sqrt{243}}.$$

Proof: From definition and by using Table 2, we obtain

$$NEU(G,x) = \sum_{uv\in E(G)} x^{\sqrt{S_u^2+S_v^2+S_uS_v}}$$

$$= nx^{\sqrt{5^2+5^2+5\times5}} + 2nx^{\sqrt{5^2+7^2+5\times7}} + 2(k-1)nx^{\sqrt{6^2+7^2+6\times7}}$$

$$+nkx^{\sqrt{7^2+9^2+7\times9}} + \frac{nk}{2}(3k-1)x^{\sqrt{9^2+9^2+9\times9}}$$

$$= nx^{\sqrt{75}} + 2nx^{\sqrt{109}} + 2(k-1)nx^{\sqrt{127}} + nkx^{\sqrt{193}} + \frac{nk}{2}(3k-1)x^{\sqrt{243}}.$$

Theorem 4.3. Let G be the graph of a nanocone $C_n[k]$. Then

$$^{m}NEU(G)=\frac{n}{\sqrt{75}}+\frac{2n}{\sqrt{109}}+\frac{2(k-1)n}{\sqrt{127}}+\frac{nk}{\sqrt{193}}+\frac{nk(3k-1)}{2\sqrt{243}}.$$

Proof: From definition and by using Table 2, we deduce

$$^{m}NEU(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{S_u^2+S_v^2+S_uS_v}}$$

$$=\frac{n}{\sqrt{5^2+5^2+5\times 5}}+\frac{2n}{\sqrt{5^2+7^2+5\times 7}}+\frac{2(k-1)n}{\sqrt{6^2+7^2+6\times 7}}$$

$$+\frac{nk}{\sqrt{7^2+9^2+7\times 9}}+\frac{nk(3k-1)}{2\sqrt{9^2+9^2+9\times 9}}$$

$$=\frac{n}{\sqrt{75}}+\frac{2n}{\sqrt{109}}+\frac{2(k-1)n}{\sqrt{127}}+\frac{nk}{\sqrt{193}}+\frac{nk(3k-1)}{2\sqrt{243}}.$$

Theorem 4.4. Let G be the graph of a nanocone $C_n[k]$. Then

$$^{m}NEU(G,x)=nx^{\frac{1}{\sqrt{75}}}+2nx^{\frac{1}{\sqrt{109}}}+2(k-1)nx^{\frac{1}{\sqrt{127}}}+nkx^{\frac{1}{\sqrt{193}}}+\frac{nk}{2}(3k-1)x^{\frac{1}{\sqrt{243}}}.$$

Proof: From definition and by using Table 2, we get

$$^{m}NEU(G,x)=\sum_{uv\in E(G)}x^{\frac{1}{\sqrt{S_u^2+S_v^2+S_uS_v}}}$$

$$=nx^{\frac{1}{\sqrt{5^2+5^2+5\times 5}}}+2nx^{\frac{1}{\sqrt{5^2+7^2+5\times 7}}}+2(k-1)nx^{\frac{1}{\sqrt{6^2+7^2+6\times 7}}}$$

$$+nkx^{\frac{1}{\sqrt{7^2+9^2+7\times 9}}}+\frac{nk}{2}(3k-1)x^{\frac{1}{\sqrt{9^2+9^2+9\times 9}}}$$

$$=nx^{\frac{1}{\sqrt{75}}}+2nx^{\frac{1}{\sqrt{109}}}+2(k-1)nx^{\frac{1}{\sqrt{127}}}+nkx^{\frac{1}{\sqrt{193}}}+\frac{nk}{2}(3k-1)x^{\frac{1}{\sqrt{243}}}.$$

5. KV EULER SOMBOR BANHATTI INDEX

Let M_u denote the product of the degrees of all vertices adjacent to a vertex u.
We introduce a new graph index defined as

$$KVEU(G)=\sum_{uv\in E(G)}\sqrt{M_u^2+M_v^2+M_uM_v}$$

which we propose to be named as KV Euler SomborBanhatti index.

Considering the KV Euler SomborBanhatti index, we introduce the KV Euler SomborBanhatti exponential of a graph G and defined it as

$$KVEU(G,x)=\sum_{uv\in E(G)}x^{\sqrt{M_u^2+M_v^2+M_uM_v}}.$$

We define the modified KV Euler SomborBanhatti index of a graph G as

$$^{m}KVEU(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{M_u^2+M_v^2+M_uM_v}}.$$

Considering the modified KV Euler SomborBanhatti index, we introduce the modified KV Euler SomborBanhatti exponential of a graph G and defined it as

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

$$ {}^{m}KVEU(G,x)=\sum_{uv\in E(G)} x^{\frac{1}{\sqrt{M_u^2+M_v^2+M_uM_v}}}. $$

Recently, some KV indices were studied in [33-36].

RESULTS FOR *POPAM* DENDRIMERS

The family of POPAM dendrimers is symbolized by $POD_2[n]$, where n is the steps of growth in this type of dendrimers. The graph of $POD_2[2]$ is shown in Figure 3.

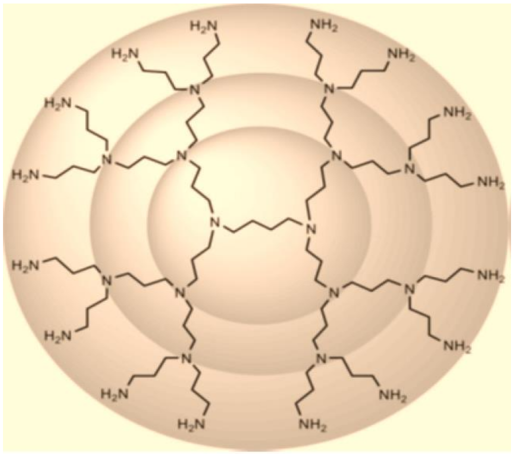

Figure 3. Graph of POD_2[2]

Let $G = POD_2[n]$. By calculation, we have $|V(G)| = 2^{n+5} - 10$ and $|E(G)| = 2^{n+5} - 11$.

Let $G = POD_2[n]$ be a POPAM dendrimer with $2^{n+5} - 11$ edges. The edge partition of $POD_2[n]$ based on the degree product of neighbors of end vertices of each edge is as follows:

$E_1 = \{ uv\in E(G) \mid M_u = M_v = 2 \}$, $|E_1| = 2^{n+2}$.
$E_2 = \{ uv\in E(G) \mid M_u = 2, M_v = 4 \}$, $|E_2| = 2^{n+2}$.
$E_3 = \{ uv\in E(G) \mid M_u = M_v= 4\}$, $|E_3| = 1$.
$E_4 = \{ uv\in E(G) \mid M_u = 4, M_v = 6 \}$, $|E_4| = 3\times2^n - 6$.
$E_5 = \{ uv\in E(G) \mid M_u = 6, M_v = 8 \}$, $|E_5| = 3\times2^n - 6$.

Theorem 5.1. Let$G$ be the graph of a POPAM dendrimer $POD_2[n]$. Then

$$ KVEU(G)=2^{n+2}\sqrt{12}+2^{n+2}\sqrt{28}+1\sqrt{48}+(3\times2^n-6)\sqrt{76}+(3\times2^n-6)\sqrt{148}. $$

Proof: From definition, we get

$$ KVEU(G)=\sum_{uv\in E(G)}\sqrt{M_u^2+M_v^2+M_uM_v} $$

$$ =2^{n+2}\sqrt{2^2+2^2+2\times2}+2^{n+2}\sqrt{2^2+4^2+2\times4}+1\sqrt{4^2+4^2+4\times4} $$

$$ +(3\times2^n-6)\sqrt{4^2+6^2+4\times6}+(3\times2^n-6)\sqrt{6^2+8^2+6\times8} $$

$$ =2^{n+2}\sqrt{12}+2^{n+2}\sqrt{28}+1\sqrt{48}+(3\times2^n-6)\sqrt{76}+(3\times2^n-6)\sqrt{148}. $$

Theorem 5.2. Let$G$ be the graph of a POPAM dendrimer $POD_2[n]$. Then

$$ KVEU(G,x)=2^{n+2}x^{\sqrt{12}}+2^{n+2}x^{\sqrt{28}}+1x^{\sqrt{48}}+(3\times2^n-6)x^{\sqrt{76}}+(3\times2^n-6)x^{\sqrt{148}}. $$

Proof: From definition, we derive

$$ KVEU(G,x)=\sum_{uv\in E(G)}x^{\sqrt{M_u^2+M_v^2+M_uM_v}} $$

$$ =2^{n+2}x^{\sqrt{2^2+2^2+2\times2}}+2^{n+2}x^{\sqrt{2^2+4^2+2\times4}}+1x^{\sqrt{4^2+4^2+4\times4}} $$

$$ +(3\times2^n-6)x^{\sqrt{4^2+6^2+4\times6}}+(3\times2^n-6)x^{\sqrt{6^2+8^2+6\times8}} $$

$$= 2^{n+2}x^{\sqrt{12}} + 2^{n+2}x^{\sqrt{28}} + 1x^{\sqrt{48}} + (3\times 2^n - 6)x^{\sqrt{76}} + (3\times 2^n - 6)x^{\sqrt{148}}.$$

Theorem 5.3. Let G be the graph of a POPAM dendrimer $POD_2[n]$. Then

$$^mKVEU(G) = \frac{2^{n+2}}{\sqrt{12}} + \frac{2^{n+2}}{\sqrt{28}} + \frac{1}{\sqrt{48}} + \frac{3\times 2^n - 6}{\sqrt{76}} + \frac{3\times 2^n - 6}{\sqrt{148}}.$$

Proof: From definition, we obtain

$$^mKVEU(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{M_u^2 + M_v^2 + M_uM_v}}$$

$$= \frac{2^{n+2}}{\sqrt{2^2+2^2+2\times 2}} + \frac{2^{n+2}}{\sqrt{2^2+4^2+2\times 4}} + \frac{1}{\sqrt{4^2+4^2+4\times 4}}$$

$$+ \frac{3\times 2^n - 6}{\sqrt{4^2+6^2+4\times 6}} + \frac{3\times 2^n - 6}{\sqrt{6^2+8^2+6\times 8}}$$

$$= \frac{2^{n+2}}{\sqrt{12}} + \frac{2^{n+2}}{\sqrt{28}} + \frac{1}{\sqrt{48}} + \frac{3\times 2^n - 6}{\sqrt{76}} + \frac{3\times 2^n - 6}{\sqrt{148}}.$$

Theorem 5.4. Let G be the graph of a POPAM dendrimer $POD_2[n]$. Then

$$^mKVEU(G,x) = 2^{n+2}x^{\frac{1}{\sqrt{12}}} + 2^{n+2}x^{\frac{1}{\sqrt{28}}} + 1x^{\frac{1}{\sqrt{48}}} + (3\times 2^n - 6)x^{\frac{1}{\sqrt{76}}} + (3\times 2^n - 6)x^{\frac{1}{\sqrt{148}}}.$$

Proof: From definition, we deduce

$$^mKVEU(G,x) = \sum_{uv\in E(G)} x^{\frac{1}{\sqrt{M_u^2+M_v^2+M_uM_v}}}$$

$$= 2^{n+2}x^{\frac{1}{\sqrt{2^2+2^2+2\times 2}}} + 2^{n+2}x^{\frac{1}{\sqrt{2^2+4^2+2\times 4}}} + 1x^{\frac{1}{\sqrt{4^2+4^2+4\times 4}}}$$

$$+ (3\times 2^n - 6)x^{\frac{1}{\sqrt{4^2+6^2+4\times 6}}} + (3\times 2^n - 6)x^{\frac{1}{\sqrt{6^2+8^2+6\times 8}}}$$

$$= 2^{n+2}x^{\frac{1}{\sqrt{12}}} + 2^{n+2}x^{\frac{1}{\sqrt{28}}} + 1x^{\frac{1}{\sqrt{48}}} + (3\times 2^n - 6)x^{\frac{1}{\sqrt{76}}} + (3\times 2^n - 6)x^{\frac{1}{\sqrt{148}}}.$$

6. DELTA EULER SOMBOR BANHATTI INDEX

The δ vertex degree was defined in [37] as

$$\delta_u = d_G(u) - \delta(G) + 1.$$

The above definition motivates us to introduce a new graph index, defined as

$$\delta EU(G) = \sum_{uv\in E(G)} \sqrt{\delta_u^2 + \delta_v^2 + \delta_u\delta_v}$$

which we propose to be named as delta Euler SomborBanhatti index.

Considering the delta Euler SomborBanhatti index, we introduce the delta Euler SomborBanhatti exponential of a graph G and defined it as

$$\delta EU(G,x) = \sum_{uv\in E(G)} x^{\sqrt{\delta_u^2+\delta_v^2+\delta_u\delta_v}}.$$

We define the modified Euler SomborBanhatti index of a graph G as

$$^m\delta EU(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{\delta_u^2 + \delta_v^2 + \delta_u\delta_v}}.$$

Considering the modified delta Euler SomborBanhatti index, we introduce the modified delta Euler SomborBanhatti exponential of a graph G and defined it as

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

$$^{m}\delta EU(G,x)=\sum_{uv\in E(G)} x^{\frac{1}{\sqrt{\delta_u^2+\delta_v^2+\delta_u\delta_v}}}.$$

Recently, some delta indices were studied in [37-45].

RESULTS FOR $HC_5C_7[p,q]$ NANOTUBES

From Figure 1, we have δ(*G*)=2. Therefore $\delta_u = d_u - \delta(G) + 1 = d_u - 1$. Thus there are two types of δ-edges as given in Table 3.

$\delta_u, \delta_v \backslash uv \in E(G)$	Number of edges
(1, 2)	$4p$
(2, 2)	$6pq - 5p$

Table 3. δ-edge partition of $HC_5C_7[p, q]$

Theorem 6.1. Let *G* be the graph of a nanotube $HC_5C_7[p, q]$. Then

$$\delta EU(G)=6\sqrt{12}\,pq+\left(4\sqrt{7}-5\sqrt{12}\right)p.$$

Proof: From definition and by using Table 3, we deduce

$$\delta EU(G)=\sum_{uv\in E(G)}\sqrt{\delta_u^2+\delta_v^2+\delta_u\delta_v}$$

$$=4p\sqrt{1^2+2^2+1\times 2}+(6pq-5p)\sqrt{2^2+2^2+2\times 2}$$

$$=6\sqrt{12}\,pq+\left(4\sqrt{7}-5\sqrt{12}\right)p.$$

Theorem 6.2. Let *G* be the graph of a nanotube $HC_5C_7[p, q]$. Then

$$\delta EU(G,x)=4px^{\sqrt{7}}+(6pq-5p)x^{\sqrt{12}}.$$

Proof: From definition and by using Table 3, we derive

$$\delta EU(G,x)=\sum_{uv\in E(G)} x^{\sqrt{\delta_u^2+\delta_v^2+\delta_u\delta_v}}$$

$$=4px^{\sqrt{1^2+2^2+1\times 2}}+(6pq-5p)x^{\sqrt{2^2+2^2+2\times 2}}$$

$$=4px^{\sqrt{7}}+(6pq-5p)x^{\sqrt{12}}.$$

Theorem 6.3. Let *G* be the graph of a nanotube $HC_5C_7[p, q]$. Then

$$^{m}\delta EU(G)=\frac{6pq}{\sqrt{12}}+\frac{4p}{\sqrt{7}}-\frac{5p}{\sqrt{12}}.$$

Proof: From definition and by using Table 3, we obtain

$$^{m}\delta EU(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{\delta_u^2+\delta_v^2+\delta_u\delta_v}}$$

$$=\frac{4p}{\sqrt{1^2+2^2+1\times 2}}+\frac{6pq-5p}{\sqrt{2^2+2^2+2\times 2}}$$

$$=\frac{6pq}{\sqrt{12}}+\frac{4p}{\sqrt{7}}-\frac{5p}{\sqrt{12}}.$$

Theorem 6.4. Let *G* be the graph of a nanotube $HC_5C_7[p, q]$. Then

$$^{m}\delta EU(G,x)=4px^{\frac{1}{\sqrt{7}}}+(6pq-5p)x^{\frac{1}{\sqrt{12}}}.$$

Proof: From definition and by using Table 3, we deduce

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

$$^{m}\delta EU(G,x)=\sum_{uv\in E(G)} x^{\frac{1}{\sqrt{\delta_u^2+\delta_v^2+\delta_u\delta_v}}}$$

$$=4px^{\frac{1}{\sqrt{1^2+2^2+1\times 2}}}+(6pq-5p)x^{\frac{1}{\sqrt{2^2+2^2+2\times 2}}}$$

$$=4px^{\frac{1}{\sqrt{7}}}+(6pq-5p)x^{\frac{1}{\sqrt{12}}}.$$

7. REVAN EULER SOMBOR BANHATTI INDEX

The Revan vertex degree [51] of a vertex u in G is defined as

$r_u = \Delta(G) + \delta(G) - d_u$.

The Revan edge connecting the Revan vertices u and v will be denoted by uv

The above definition motivates us to introduce a new index, defined as

$$REU(G)=\sum_{uv\in E(G)}\sqrt{r_u^2+r_v^2+r_ur_v}$$

which we propose to be named as Revan Euler SomborBanhatti index.

Considering the Revan Euler SomborBanhatti index, we propose the Revan Euler SomborBanhatti exponential of a graph G and defined it as

$$REU(G,x)=\sum_{uv\in E(G)} x^{\sqrt{r_u^2+r_v^2+r_ur_v}}.$$

We define the modified Revan Euler SomborBanhatti index of a graph G as

$$^{m}REU(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{r_u^2+r_v^2+r_ur_v}}.$$

Considering the modified Revan Euler SomborBanhatti index, we introduce the modified Revan Euler SomborBanhatti exponential of a graph G and defined it as

$$^{m}REU(G,x)=\sum_{uv\in E(G)} x^{\frac{1}{\sqrt{r_u^2+r_v^2+r_ur_v}}}.$$

Recently, some Revan indices were studied in [46-59].

RESULTS FOR OXIDE NETWORKS

An oxide network of n is symbolized by OX_n. These networks are of vital importance in the study of silicate networks. An oxide network of dimension five is shown in Figure 4.

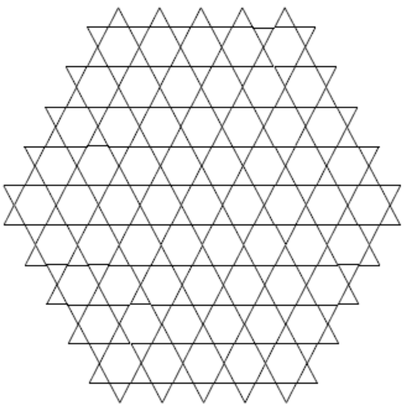

Figure 4. 5-dimensional oxide network

Let G be the graph of oxide network OX_n with $9n^2 + 3n$ vertices and $18n^2$ edges. From Figure 4, it is easy to see that the vertices of OX_n are either of degree 2 or 4. Thus $\Delta(G) = 4$, $\delta(G) = 2$. In G, there are two types of edges as follows:

$E_1 = \{uv\in E(G)|\ d_u = 2,\ d_v = 4\}$, $|E_1| = 12n$.

$E_2 = \{uv\in E(G)|\ d_u = d_v = 4\}$, $|E_2| = 18n^2 - 12n$.

We have $r_u = \Delta(G) + \delta(G) - d_u = 6 - d_u$.

$RE_1 = \{uv\in E(G) \mid r_u = 4,\ r_v = 2\}$, $|RE_1| = 12n$.

$RE_2 = \{uv\in E(G) \mid r_u = r_v = 2\}$, $|RE_2| = 18n^2 - 12n$.

Theorem 7.1. Let G be the graph of a oxide network OX_n. Then

$$REU(G) = 18\sqrt{12}n^2 + 12\sqrt{28}n - 12\sqrt{12}n.$$

Proof: From definition, we deduce

$$REU(G) = \sum_{uv\in E(G)} \sqrt{r_u^2 + r_v^2 + r_u r_v}$$

$$= 12n\sqrt{4^2 + 2^2 + 4\times 2} + (18n^2 - 12n)\sqrt{2^2 + 2^2 + 2\times 2}$$

$$= 18\sqrt{12}n^2 + 12\sqrt{28}n - 12\sqrt{12}n.$$

Theorem 7.2. Let G be the graph of a nanotube $HC_5C_7[p, q]$. Then

$$REU(G, x) = 12nx^{\sqrt{28}} + (18n^2 - 12n)x^{\sqrt{12}}.$$

Proof: From definition, we get

$$REU(G, x) = \sum_{uv\in E(G)} x^{\sqrt{r_u^2 + r_v^2 + r_u r_v}}$$

$$= 12nx^{\sqrt{4^2+2^2+4\times 2}} + (18n^2 - 12n)x^{\sqrt{2^2+2^2+2\times 2}}$$

$$= 12nx^{\sqrt{28}} + (18n^2 - 12n)x^{\sqrt{12}}.$$

Theorem 7.3. Let G be the graph of a nanotube $HC_5C_7[p, q]$. Then

$$^{m}REU(G) = \frac{18n^2}{\sqrt{12}} + \frac{12n}{\sqrt{28}} - \frac{12n}{\sqrt{12}}.$$

Proof: From definition, we deduce

$$^{m}REU(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{r_u^2 + r_v^2 + r_u r_v}}$$

$$= \frac{12n}{\sqrt{4^2 + 2^2 + 4\times 2}} + \frac{18n^2 - 12n}{\sqrt{2^2 + 2^2 + 2\times 2}}$$

$$= \frac{18n^2}{\sqrt{12}} + \frac{12n}{\sqrt{28}} - \frac{12n}{\sqrt{12}}.$$

Theorem 7.4. Let G be the graph of a nanotube $HC_5C_7[p, q]$. Then

$$^{m}REU(G, x) = 12nx^{\frac{1}{\sqrt{28}}} + (18n^2 - 12n)x^{\frac{1}{\sqrt{12}}}.$$

Proof: From definition, we obtain

$$^{m}REU(G, x) = \sum_{uv\in E(G)} x^{\frac{1}{\sqrt{r_u^2 + r_v^2 + r_u r_v}}}$$

$$= 12nx^{\frac{1}{\sqrt{4^2+2^2+4\times 2}}} + (18n^2 - 12n)x^{\frac{1}{\sqrt{2^2+2^2+2\times 2}}}$$

$$= 12nx^{\frac{1}{\sqrt{28}}} + (18n^2 - 12n)x^{\frac{1}{\sqrt{12}}}.$$

8. REVERSE EULER SOMBOR INDEX

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

The reverse vertex degree of a vertex v in G is defined as

$$c_u = \Delta(G) - d_u+1.$$

The reverse edge connecting the reverse vertices u and v will be denoted by uv.

The reverse Euler Sombor index [ ] of a graph G is defined as

$$CEU(G) = \sum_{uv\in E(G)} \sqrt{c_u^2 + c_v^2 + c_u c_v}.$$

Considering the reverse Euler Sombor index, we introduce the reverse Euler Sombor exponential of a graph G and defined it as

$$CEU(G,x) = \sum_{uv\in E(G)} x^{\sqrt{c_u^2 + c_v^2 + c_u c_v}}.$$

We define the modified reverse Euler Sombor index of a graph G as

$${}^{m}CEU(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{c_u^2 + c_v^2 + c_u c_v}}.$$

Considering the modified reverse Euler Sombor index, we introduce the modified reverse Euler Sombor exponential of a graph G and defined it as

$${}^{m}CEU(G,x) = \sum_{uv\in E(G)} x^{\frac{1}{\sqrt{c_u^2 + c_v^2 + c_u c_v}}}.$$

Recently, some reverse indices were studied in [60-71].

RESULTS FOR HONEYCOMB NETWORKS

Honeycomb networks are useful in Computer Graphics and Chemistry. A honeycomb network of dimension n is denoted by HC_n, where n is the number of hexagons between central and boundary hexagon. A 4-dimensional honeycomb network is shown in Figure 5.

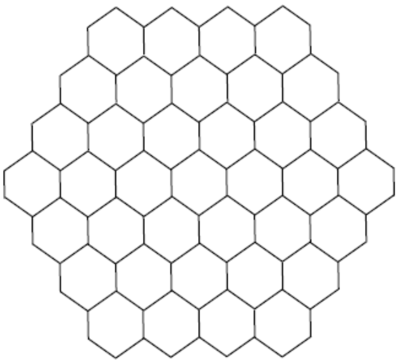

Figure 5. A 4-dimensional honeycomb network

Let G be the graph of a honeycomb network HC_n. From Figure 5, it is easy to see that $\Delta(G) = 3$. Therefore $c_u = \Delta(G) - d_G(u) + 1 = 4 - d_G(u)$. By calculation, we obtain that G has $6n^2$ vertices and $9n^2-3n$ edges. In G, by algebraic method, there are three types of edges based on the degree of end vertices of each edge as follows:

$E_{22} = \{uv\in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_{22}| = 6$.

$E_{23} = \{uv\in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_{23}| = 12n - 12$.

$E_{33} = \{uv\in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_{33}| = 9n^2 - 15n + 6$.

Thus there are three types of reverse edges as given in Table 4.

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

$c_u, c_v \setminus uv \in E(G)$	Number of edges
(2, 2)	6
(2, 1)	12n – 12
(1, 1)	$9n^2 – 15n + 6$

Table 4. Reverse edge partition of HC_n

Theorem 8.1. Let G be the graph of a honeycomb network HC_n. Then

$$CEU(G) = 9\sqrt{3}n^2 + 12\sqrt{7}n - 15\sqrt{3}n + 6\sqrt{12} - 12\sqrt{7} + 6\sqrt{3}.$$

Proof: From definition and by using Table 4, we deduce

$$CEU(G) = \sum_{uv\in E(G)} \sqrt{c_u^2 + c_v^2 + c_u c_v}$$

$$= 6\sqrt{2^2 + 2^2 + 2\times 2} + (12n-12)\sqrt{2^2 + 1^2 + 2\times 1}$$

$$+ (9n^2 - 15n + 6)\sqrt{1^2 + 1^2 + 1\times 1}$$

$$= 9\sqrt{3}n^2 + 12\sqrt{7}n - 15\sqrt{3}n + 6\sqrt{12} - 12\sqrt{7} + 6\sqrt{3}.$$

Theorem 8.2. Let G be the graph of a honeycomb network HC_n. Then

$$CEU(G,x) = 6x^{\sqrt{12}} + (12n-12)x^{\sqrt{7}} + (9n^2 - 15n + 6)x^{\sqrt{3}}.$$

Proof: From definition and by using Table 4, we obtain

$$CEU(G,x) = \sum_{uv\in E(G)} x^{\sqrt{c_u^2 + c_v^2 + c_u c_v}}$$

$$= 6x^{\sqrt{2^2+2^2+2\times 2}} + (12n-12)x^{\sqrt{2^2+1^2+2\times 1}} + (9n^2 - 15n + 6)x^{\sqrt{1^2+1^2+1\times 1}}$$

$$= 6x^{\sqrt{12}} + (12n-12)x^{\sqrt{7}} + (9n^2 - 15n + 6)x^{\sqrt{3}}.$$

Theorem 8.3. Let G be the graph of a honeycomb network HC_n. Then

$$^{m}CEU(G) = \frac{9n^2}{\sqrt{3}} + \frac{12n}{\sqrt{7}} - \frac{15n}{\sqrt{3}} + \frac{6}{\sqrt{12}} - \frac{12}{\sqrt{7}} + \frac{6}{\sqrt{3}}.$$

Proof: From definition and by using Table 4, we deduce

$$^{m}CEU(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{c_u^2 + c_v^2 + c_u c_v}}$$

$$= \frac{6}{\sqrt{2^2 + 2^2 + 2\times 2}} + \frac{12n-12}{\sqrt{2^2 + 1^2 + 2\times 1}} + \frac{9n^2 - 15n + 6}{\sqrt{1^2 + 1^2 + 1\times 1}}$$

$$= \frac{9n^2}{\sqrt{3}} + \frac{12n}{\sqrt{7}} - \frac{15n}{\sqrt{3}} + \frac{6}{\sqrt{12}} - \frac{12}{\sqrt{7}} + \frac{6}{\sqrt{3}}.$$

Theorem 8.4. Let G be the graph of a honeycomb network HC_n. Then

$$^{m}CEU(G,x) = 6x^{\frac{1}{\sqrt{122}}} + (12n-12)x^{\frac{1}{\sqrt{7}}} + (9n^2 - 15n + 6)x^{\frac{1}{\sqrt{3}}}.$$

Proof: From definition and by using Table 4, we get

$$^{m}CEU(G,x) = \sum_{uv\in E(G)} x^{\frac{1}{\sqrt{c_u^2 + c_v^2 + c_u c_v}}}$$

$$= 6x^{\frac{1}{\sqrt{2^2+2^2+2\times 2}}} + (12n-12)x^{\frac{1}{\sqrt{2^2+1^2+2\times 1}}} + (9n^2 - 15n + 6)x^{\frac{1}{\sqrt{1^2+1^2+1\times 1}}}$$

$$= 6x^{\frac{1}{\sqrt{122}}} + (12n - 12)x^{\frac{1}{\sqrt{7}}} + (9n^2 - 15n + 6)x^{\frac{1}{\sqrt{3}}}.$$

9. LEAP EULER SOMBOR BANHATTI INDEX

The distance $d(u,v)$ between any two vertices u and v of G is the number of edges in a shortest path connecting the vertices u and v. For a positive integer k, and a vertex v in G is the open neighborhood of v in G is defined as $N_k(v/G)=\{u\in V(G) : d(u, v) = k\}$. The k-distance degree of a vertex v in G is the number of k neighbours of v in G and it is denoted by $d_k(v)$, see [86].

We introduce a new topological index, defined as

$$LEU(G) = \sum_{uv\in E(G)} \sqrt{d_2(u)^2 + d_2(v)^2 + d_2(u)d_2(v)}$$

which we propose to be named as leap Euler SomborBanhatti index.

Considering the leap Euler Sombor index, we introduce the leap Euler SomborBanhatti exponential of a graph G and defined it as

$$LEU(G,x) = \sum_{uv\in E(G)} x^{\sqrt{d_2(u)^2 + d_2(v)^2 + d_2(u)d_2(v)}}.$$

We define the modified leap Euler SomborBanhatti index of a graph G as

$${}^m LEU(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_2(u)^2 + d_2(v)^2 + d_2(u)d_2(v)}}.$$

Considering the modified leap Euler SomborBanhatti index, we introduce the modified leap Euler SomborBanhatti exponential of a graph G and defined it as

$${}^m LEU(G,x) = \sum_{uv\in E(G)} x^{\frac{1}{\sqrt{d_2(u)^2 + d_2(v)^2 + d_2(u)d_2(v)}}}.$$

Recently, some leap indices were studied in [72-86].

RESULTS FOR WHEEL GRAPHS

The wheel W_n is defined to be the join of cycle C_n and complete graph K_1. The wheel W_n has n+1vertices and 2n edges, see Figure 6. The vertex K_1 is called apex and the vertices of C_n are called rim vertices.

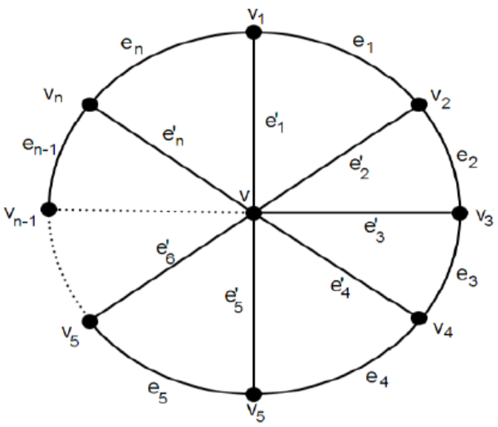

Figure 6. Wheel W_n

In W_n, there are two types of the 2-distance degree of edges as follows:

$E_1 = \{uv\in E(W_n) \mid d_2(u) = 0, d_2(v) = n - 3\}$, $|E_1| = n$.

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

$E_2 = \{uv\in E(W_n) \mid d_2(u) = d_2(v) = n - 3\}$, $|E_2| = n$.

Theorem 9.1. Let $G= W_n$ be the wheel graph. Then

$$LEU(G)=\left(1+\sqrt{3}\right)n(n-3).$$

Proof: From definition, we obtain

$$LEU(G)=\sum_{uv\in E(G)}\sqrt{d_2(u)^2+d_2(v)^2+d_2(u)d_2(v)}$$

$$=n\sqrt{0^2+(n-3)^2+0\times(n-3)}+n\sqrt{(n-3)^2+(n-3)^2+(n-3)\times(n-3)}$$

$$=\left(1+\sqrt{3}\right)n(n-3).$$

Theorem 9.2. Let $G= W_n$ be the wheel graph. Then

$$LEU(G,x)=nx^{n-3}+nx^{\sqrt{3}(n-3)}.$$

Proof: From definition, we deduce

$$LEU(G,x)=\sum_{uv\in E(G)}x^{\sqrt{d_2(u)^2+d_2(v)^2+d_2(u)d_2(v)}}$$

$$=nx^{\sqrt{0^2+(n-3)^2+0\times(n-3)}}+nx^{\sqrt{(n-3)^2+(n-3)^2+(n-3)\times(n-3)}}$$

$$=nx^{n-3}+nx^{\sqrt{3}(n-3)}.$$

Theorem 9.3. Let $G= W_n$ be the wheel graph. Then

$$^{m}LEU(G)=\frac{n}{n-3}+\frac{n}{\sqrt{3}(n-3)}.$$

Proof: From definition, we deduce

$$^{m}LEU(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_2(u)^2+d_2(v)^2+d_2(u)d_2(v)}}$$

$$=\frac{n}{\sqrt{0^2+(n-3)^2+0\times(n-3)}}+\frac{n}{\sqrt{(n-3)^2+(n-3)^2+(n-3)\times(n-3)}}$$

$$=\frac{n}{n-3}+\frac{n}{\sqrt{3}(n-3)}.$$

Theorem 9.4. Let $G= W_n$ be the wheel graph. Then

$$^{m}LEU(G,x)=nx^{\frac{1}{n-3}}+nx^{\frac{1}{\sqrt{3}(n-3)}}.$$

Proof: From definition, we get

$$^{m}LEU(G,x)=\sum_{uv\in E(G)}x^{\frac{1}{\sqrt{d_2(u)^2+d_2(v)^2+d_2(u)d_2(v)}}}$$

$$=nx^{\frac{1}{\sqrt{0^2+(n-3)^2+0\times(n-3)}}}+nx^{\frac{1}{\sqrt{(n-3)^2+(n-3)^2+(n-3)\times(n-3)}}}$$

$$=nx^{\frac{1}{n-3}}+nx^{\frac{1}{\sqrt{3}(n-3)}}.$$

10. STATUS EULER SOMBOR BANHATTI INDEX

The distance $d(u, v)$ between any two vertices u and v is the length of shortest path connecting u and v. The status $\sigma(u)$ [87] of a vertex u in G is the sum of distances of all other vertices from u in G.

We put forward a new topological index, defined as

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

$$SEU(G)=\sum_{uv\in E(G)}\sqrt{\sigma(u)^2+\sigma(v)^2+\sigma(u)\sigma(v)}$$

which we propose to be named as status Euler SomborBanhatti index.

Considering the status Euler SomborBanhatti index, we introduce the status Euler SomborBanhatti exponential of a graph G and defined it as

$$SEU(G,x)=\sum_{uv\in E(G)}x^{\sqrt{\sigma(u)^2+\sigma(v)^2+\sigma(u)\sigma(v)}}.$$

We define the modified status Euler SomborBanhatti index of a graph G as

$${}^{m}SEU(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{\sigma(u)^2+\sigma(v)^2+\sigma(u)\sigma(v)}}.$$

Considering the modified status Euler SomborBanhatti index, we introduce the modified status Euler SomborBanhatti exponential of a graph G and defined it as

$${}^{m}SEU(G,x)=\sum_{uv\in E(G)}x^{\frac{1}{\sqrt{\sigma(u)^2+\sigma(v)^2+\sigma(u)\sigma(v)}}}.$$

Recently, some status indices were studied in [87-101].

RESULTS FOR FRIENDSHIP GRAPHS

A friendship graph F_n, $n \geq 2$, is a graph that can be constructed by joining n copies of C_3 with a common vertex. A graph F_4 is presented in Figure 7.

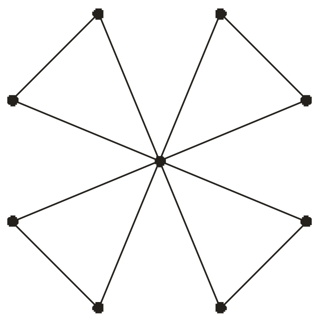

Figure 7. Friendship graph F_4

Let F_n be a friendship graph with $2n+1$ vertices and $3n$ edges. By calculation, we obtain that there are two types of edges as follows:

$E_1=\{uv\in E(F_n)\mid d_{F_n}(u)=d_{F_n}(v)=2\}$, $|E_1| = n$.

$E_2=\{uv\in E(F_n)\mid d_{F_n}(u)=2, d_{F_n}(v)=2n\}$, $|E_2| = 2n$.

Therefore, in F_n, there are two types of status edges as given in Table 5.

$\sigma(u), \sigma(v) \setminus uv \in E(F_n)$	Number of edges
$(4n-2, 4n-2)$	n
$(2n, 4n-2)$	$2n$

Table 5. Status edge partition of F_n

Theorem 10.1. Let $G = F_n$ be the friendship graph. Then

$$SEU(G) = \sqrt{3}n(4n-2) + 2n\sqrt{(2n)^2 + (4n-2)^2 + (2n)\times(4n-2)}.$$

Proof: From definition and by using Table 5, we deduce

$$SEU(G) = \sum_{uv\in E(G)} \sqrt{\sigma(u)^2 + \sigma(v)^2 + \sigma(u)\sigma(v)}$$

$$= n\sqrt{(4n-2)^2 + (4n-2)^2 + (4n-2)\times(4n-2)}$$

$$+2n\sqrt{(2n)^2 + (4n-2)^2 + (2n)\times(4n-2)}$$

$$= \sqrt{3}n(4n-2) + 2n\sqrt{(2n)^2 + (4n-2)^2 + (2n)\times(4n-2)}.$$

Theorem 10.2. Let $G = F_n$ be the friendship graph. Then

$$SEU(G,x) = nx^{\sqrt{3}(4n-2)} + 2nx^{\sqrt{(2n)^2+(4n-2)^2+(2n)\times(4n-2)}}.$$

Proof: From definition and by using Table 5, we deduce

$$SEU(G,x) = \sum_{uv\in E(G)} x^{\sqrt{\sigma(u)^2+\sigma(v)^2+\sigma(u)\sigma(v)}}$$

$$= nx^{\sqrt{(4n-2)^2+(4n-2)^2+(4n-2)\times(4n-2)}} + 2nx^{\sqrt{(2n)^2+(4n-2)^2+(2n)\times(4n-2)}}$$

$$= nx^{\sqrt{3}(4n-2)} + 2nx^{\sqrt{(2n)^2+(4n-2)^2+(2n)\times(4n-2)}}.$$

Theorem 10.3. Let $G = F_n$ be the friendship graph. Then

$$^{m}SEU(G) = \frac{n}{\sqrt{3}(4n-2)}.+\frac{2n}{\sqrt{(2n)^2 + (4n-2)^2 + (2n)\times(4n-2)}}.$$

Proof: From definition and by using Table 5, we obtain

$$^{m}SEU(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{\sigma(u)^2 + \sigma(v)^2 + \sigma(u)\sigma(v)}}$$

$$= \frac{n}{\sqrt{(4n-2)^2 + (4n-2)^2 + (4n-2)\times(4n-2)}}$$

$$+\frac{2n}{\sqrt{(2n)^2 + (4n-2)^2 + (2n)\times(4n-2)}}$$

$$= \frac{n}{\sqrt{3}(4n-2)}.+\frac{2n}{\sqrt{(2n)^2 + (4n-2)^2 + (2n)\times(4n-2)}}.$$

Theorem 10.4. Let $G = F_n$ be the friendship graph. Then

$$^{m}SEU(G,x) = nx^{\frac{1}{\sqrt{3}(4n-2)}} + 2nx^{\frac{1}{\sqrt{(2n)^2+(4n-2)^2+(2n)\times(4n-2)}}}.$$

Proof: From definition and by using Table 5, we get

$$^{m}SEU(G,x) = \sum_{uv\in E(G)} x^{\frac{1}{\sqrt{\sigma(u)^2+\sigma(v)^2+\sigma(u)\sigma(v)}}}$$

$$= nx^{\frac{1}{\sqrt{(4n-2)^2+(4n-2)^2+(4n-2)\times(4n-2)}}} + 2nx^{\frac{1}{\sqrt{(2n)^2+(4n-2)^2+(2n)\times(4n-2)}}}$$

$$= nx^{\frac{1}{\sqrt{3}(4n-2)}} + 2nx^{\frac{1}{\sqrt{(2n)^2+(4n-2)^2+(2n)\times(4n-2)}}}.$$

11. ve-DEGREE EULER SOMBOR BANHATTI INDEX

The ve-degree $d_{ve}(u)$ [102] of a vertex u in a graph G is the number of different edges that incident to any vertex from the closed neighborhood of u.

We introduce a new topological index, defined as

$$VEEU(G) = \sum_{uv \in E(G)} \sqrt{d_{ve}(u)^2 + d_{ve}(v)^2 + d_{ve}(u)d_{ve}(v)}$$

which we propose to be named as ve-degree Euler SomborBanhatti index.

Considering the ve-degree Euler SomborBanhatti index, we introduce the ve-degree Euler SomborBanhatti exponential of a graph G and defined it as

$$VEEU(G, x) = \sum_{uv \in E(G)} x^{\sqrt{d_{ve}(u)^2 + d_{ve}(v)^2 + d_{ve}(u)d_{ve}(v)}}.$$

We define the modified ve-degree Euler SomborBanhatti index of a graph G as

$${}^{m}VEEU(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_{ve}(u)^2 + d_{ve}(v)^2 + d_{ve}(u)d_{ve}(v)}}.$$

Considering the modified ve-degree Euler SomborBanhatti index, we introduce the modified ve-degree Euler SomborBanhatti exponential of a graph G and defined it as

$${}^{m}VEEU(G, x) = \sum_{uv \in E(G)} x^{\frac{1}{\sqrt{d_{ve}(u)^2 + d_{ve}(v)^2 + d_{ve}(u)d_{ve}(v)}}}.$$

Recently, some ve-degree indices were studied in [102-113].

RESULTS FOR DOMINATING OXIDE NEETWORKS

The family of dominating oxide networks is symbolized by $DOX(n)$. The molecular structure of a dominating oxide network is presented in Figure 8.

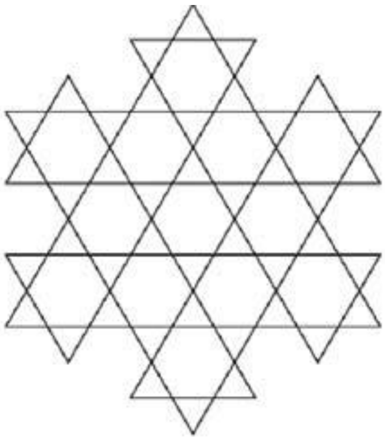

Figure 8. The structure of a dominating oxide network

Ediz obtained the ve-degree partition of the end vertices of edges for dominating oxide networks in Table 6.

$(d_{ve}, d_{ve}) \backslash uv \in E(G)$	Number of edges
(7, 10)	$12n$
(7, 12)	$12n-12$
(10, 10)	6
(10, 12)	$12n-12$
(12, 14)	$24n-24$
(14, 14)	$54n^2-114n+60$

Table 6. The ve-degree of the end vertices of edges for *DOX* networks

Theorem 11.1. Let G be the graph of $DOX(n)$. Then

$$VEEU(G) = 2n\sqrt{219} + (12n-12)\sqrt{277} + 6\sqrt{300} + (12n-12)\sqrt{364} + (24n-24)\sqrt{508} + (54n^2-114n+60)\sqrt{588}.$$

Proof: From definition and by using Table 6, we deduce

$$VEEU(G) = \sum_{uv\in E(G)} \sqrt{d_{ve}(u)^2 + d_{ve}(v)^2 + d_{ve}(u)d_{ve}(v)}$$

$$= 2n\sqrt{7^2+10^2+7\times 10} + (12n-12)\sqrt{7^2+12^2+7\times 12} + 6\sqrt{10^2+10^2+10\times 10}$$
$$+ (12n-12)\sqrt{10^2+12^2+10\times 12} + (24n-24)\sqrt{12^2+14^2+12\times 14}$$
$$+ (54n^2-114n+60)\sqrt{14^2+14^2+14\times 14}$$
$$= 2n\sqrt{219} + (12n-12)\sqrt{277} + 6\sqrt{300} + (12n-12)\sqrt{364}$$
$$+ (24n-24)\sqrt{508} + (54n^2-114n+60)\sqrt{588}.$$

Theorem 11.2. Let G be the graph of $DOX(n)$. Then

$$VEEU(G,x) = 2nx^{\sqrt{219}} + (12n-12)x^{\sqrt{277}} + 6x^{\sqrt{300}} + (12n-12)x^{\sqrt{364}} + (24n-24)x^{\sqrt{508}} + (54n^2-114n+60)x^{\sqrt{588}}.$$

Proof: From definition and by using Table 6, we derive

$$VEEU(G,x) = \sum_{uv\in E(G)} x^{\sqrt{d_{ve}(u)^2 + d_{ve}(v)^2 + d_{ve}(u)d_{ve}(v)}}$$

$$= 2nx^{\sqrt{7^2+10^2+7\times 10}} + (12n-12)x^{\sqrt{7^2+12^2+7\times 12}} + 6x^{\sqrt{10^2+10^2+10\times 10}}$$
$$+ (12n-12)x^{\sqrt{10^2+12^2+10\times 12}} + (24n-24)x^{\sqrt{12^2+14^2+12\times 14}}$$
$$+ (54n^2-114n+60)x^{\sqrt{14^2+14^2+14\times 14}}$$
$$= 2nx^{\sqrt{219}} + (12n-12)x^{\sqrt{277}} + 6x^{\sqrt{300}} + (12n-12)x^{\sqrt{364}}$$
$$+ (24n-24)x^{\sqrt{508}} + (54n^2-114n+60)x^{\sqrt{588}}.$$

Theorem 11.3. Let G be the graph of $DOX(n)$. Then

$$^{m}VEEU(G) = \frac{2n}{\sqrt{219}} + \frac{12n-12}{\sqrt{277}} + \frac{6}{\sqrt{300}} + \frac{12n-12}{\sqrt{364}} + \frac{24n-24}{\sqrt{508}} + \frac{54n^2-114n+60}{\sqrt{588}}.$$

Proof: From definition and by using Table 6, we obtain

$$^{m}VEEU(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_{ve}(u)^2 + d_{ve}(v)^2 + d_{ve}(u)d_{ve}(v)}}$$

$$= \frac{2n}{\sqrt{7^2+10^2+7\times 10}} + \frac{12n-12}{\sqrt{7^2+12^2+7\times 12}} + \frac{6}{\sqrt{10^2+10^2+10\times 10}}$$
$$+ \frac{12n-12}{\sqrt{10^2+12^2+10\times 12}} + \frac{24n-24}{\sqrt{12^2+14^2+12\times 14}} + \frac{54n^2-114n+60}{\sqrt{14^2+14^2+14\times 14}}$$
$$= \frac{2n}{\sqrt{219}} + \frac{12n-12}{\sqrt{277}} + \frac{6}{\sqrt{300}} + \frac{12n-12}{\sqrt{364}} + \frac{24n-24}{\sqrt{508}} + \frac{54n^2-114n+60}{\sqrt{588}}.$$

Theorem 11.4. Let G be the graph of $DOX(n)$. Then

$$^{m}VEEU(G,x) = 2nx^{\frac{1}{\sqrt{219}}} + (12n-12)x^{\frac{1}{\sqrt{277}}} + 6x^{\frac{1}{\sqrt{300}}} + (12n-12)x^{\frac{1}{\sqrt{364}}} + (24n-24)x^{\frac{1}{\sqrt{508}}} + (54n^2-114n+60)x^{\frac{1}{\sqrt{588}}}.$$

Proof: From definition and by using Table 6, we deduce

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

$$^{m}VEEU(G,x)=\sum_{uv\in E(G)} x^{\frac{1}{\sqrt{d_{ve}(u)^2+d_{ve}(v)^2+d_{ve}(u)d_{ve}(v)}}}$$

$$=2nx^{\frac{1}{\sqrt{7^2+10^2+7\times 10}}}+(12n-12)x^{\frac{1}{\sqrt{7^2+12^2+7\times 12}}}+6x^{\frac{1}{\sqrt{10^2+10^2+10\times 10}}}$$

$$+(12n-12)x^{\frac{1}{\sqrt{10^2+12^2+10\times 12}}}+(24n-24)x^{\frac{1}{\sqrt{12^2+14^2+12\times 14}}}$$

$$+\left(54n^2-114n+60\right)x^{\frac{1}{\sqrt{14^2+14^2+14\times 14}}}$$

$$=2nx^{\frac{1}{\sqrt{219}}}+(12n-12)x^{\frac{1}{\sqrt{277}}}+6x^{\frac{1}{\sqrt{300}}}+(12n-12)x^{\frac{1}{\sqrt{364}}}$$

$$+(24n-24)x^{\frac{1}{\sqrt{508}}}+\left(54n^2-114n+60\right)x^{\frac{1}{\sqrt{588}}}.$$

12. KULLI-BASSAVA EULER SOMBOR BANHATTI INDEX

The degree of an edge $e = uv$ in G is defined by $d_G(e)= d_u+ d_v – 2$. Let $S_e(u)$ [114] denote the sum of degrees of all edges incident to a vertex u.
We propose a new topological index, defined as

$$KBEU(G)=\sum_{uv\in E(G)}\sqrt{S_e(u)^2+S_e(v)^2+S_e(u)S_e(v)}$$

which we propose to be named as Kulli-Basava Euler SomborBanhatti index.
Considering the Kulli-Basava Euler SomborBanhatti index, we introduce the Kulli-Basava Euler SomborBanhatti exponential of a graph G and defined it as

$$KBEU(G,x)=\sum_{uv\in E(G)} x^{\sqrt{S_e(u)^2+S_e(v)^2+S_e(u)S_e(v)}}.$$

We define the modified Kulli-Basava Euler SomborBanhatti index of a graph G as

$$^{m}KBEU(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{S_e(u)^2+S_e(v)^2+S_e(u)S_e(v)}}.$$

Considering the modified Kulli-Basava Euler SomborBanhatti index, we introduce the modified Kulli-Basava Euler SomborBanhatti exponential of a graph G and defined it as

$$^{m}KBEU(G,x)=\sum_{uv\in E(G)} x^{\frac{1}{\sqrt{S_e(u)^2+S_e(v)^2+S_e(u)S_e(v)}}}.$$

Recently, some Kulli-Basava indices were studied in [114-123].

RESULTS FOR WHEEL GRAPHS

Recall, a wheel W_n is the join of K_1 and C_n. Clearly W_n has n+1vertices and 2n edges.

RESEARCHERID
THOMSON REUTERS

[KULLI , 13(6): June, 2024]
ICTM Value: 3.00

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

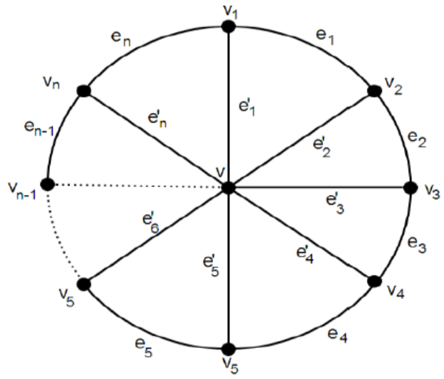

Figure 9. Wheel W_n

Let W_n be a wheel with n+1vertices and 2n edges, $n\geq 3$. Then

$E_1 = \{uv\in E(W_n) \mid S_e(u) = n+9, (S_e(v) = n(n+1)\}$, $|E_1| = n$

$E_2 = \{uv\in E(W_n) \mid S_e(u) = n+9, (S_e(v) = n + 9)\}$, $|E_2| = n$.

Theorem 12.1. Let$G= W_n$ be the wheel graph. Then

$$KBEU(G) = n\sqrt{(n+9)^2+(n^2+n)^2+(n+9)(n^2+n)} + \sqrt{3}n(n+9).$$

Proof: From definition, we deduce

$$KBEU(G) = \sum_{uv\in E(G)} \sqrt{S_e(u)^2+S_e(v)^2+S_e(u)S_e(v)}$$

$$= n\sqrt{(n+9)^2+(n^2+n)^2+(n+9)(n^2+n)}$$

$$+n\sqrt{(n+9)^2+(n+9)^2+(n+9)\times(n+9)}$$

$$= n\sqrt{(n+9)^2+(n^2+n)^2+(n+9)(n^2+n)} + \sqrt{3}n(n+9).$$

Theorem 12.2. Let $G= W_n$ be the wheel graph. Then

$$KBEU(G,x) = nx^{\sqrt{(n+9)^2+(n^2+n)^2+(n+9)(n^2+n)}} + nx^{\sqrt{3}(n+9)}.$$

Proof: From definition, we deduce

$$KBEU(G,x) = \sum_{uv\in E(G)} x^{\sqrt{S_e(u)^2+S_e(v)^2+S_e(u)S_e(v)}}$$

$$= nx^{\sqrt{(n+9)^2+(n^2+n)^2+(n+9)(n^2+n)}} + nx^{\sqrt{(n+9)^2+(n+9)^2+(n+9)\times(n+9)}}$$

$$= nx^{\sqrt{(n+9)^2+(n^2+n)^2+(n+9)(n^2+n)}} + nx^{\sqrt{3}(n+9)}.$$

Theorem 12.3. Let $G= W_n$ be the wheel graph. Then

$${}^{m}KBEU(G) = \frac{n}{\sqrt{(n+9)^2+(n^2+n)^2+(n+9)(n^2+n)}} + \frac{n}{\sqrt{3}(n+9)}.$$

Proof: From definition, we deduce

$${}^{m}KBEU(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{S_e(u)^2+S_e(v)^2+S_e(u)S_e(v)}}$$

$$= \frac{n}{\sqrt{(n+9)^2+(n^2+n)^2+(n+9)(n^2+n)}}$$

$$+ \frac{n}{\sqrt{(n+9)^2+(n+9)^2+(n+9)\times(n+9)}}$$

$$= \frac{n}{\sqrt{(n+9)^2 + (n^2+n)^2 + (n+9)(n^2+n)}} + \frac{n}{\sqrt{3}(n+9)}.$$

Theorem 12.4. Let $G= W_n$ be the wheel graph. Then

$${}^m KBEU(G,x) = nx^{\frac{1}{\sqrt{(n+9)^2+(n^2+n)^2+(n+9)(n^2+n)}}} + nx^{\frac{1}{\sqrt{3}(n+9)}}.$$

Proof: From definition, we deduce

$${}^m KBEU(G,x) = \sum_{uv\in E(G)} x^{\frac{1}{\sqrt{S_e(u)^2+S_e(v)^2+S_e(u)S_e(v)}}}$$

$$= nx^{\frac{1}{\sqrt{(n+9)^2+(n^2+n)^2+(n+9)(n^2+n)}}} + nx^{\frac{1}{\sqrt{(n+9)^2+(n+9)^2+(n+9)\times(n+9)}}}$$

$$= nx^{\frac{1}{\sqrt{(n+9)^2+(n^2+n)^2+(n+9)(n^2+n)}}} + nx^{\frac{1}{\sqrt{3}(n+9)}}.$$

13. DOMINATION EULER SOMBOR BANHATTI INDEX

The domination degree $d_d(u)$ [124] of a vertex u in a graph G is defined as the number of minimal dominating sets of Gwhich contains u.

We introduce a new topological index, defined as

$$DEU(G) = \sum_{uv\in E(G)} \sqrt{d_d(u)^2 + d_d(v)^2 + d_d(u)d_d(v)}$$

which we propose to be named as domination Euler Sombor Banhatti index.

Considering the domination Euler Sombor Banhatti index, we introduce the domination Euler SomborBanhatti exponential of a graph G and defined it as

$$DEU(G,x) = \sum_{uv\in E(G)} x^{\sqrt{d_d(u)^2+d_d(v)^2+d_d(u)d_d(v)}}.$$

We define the modified domination Euler SomborBanhatti index of a graph G as

$${}^m DEU(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_d(u)^2 + d_d(v)^2 + d_d(u)d_d(v)}}.$$

Considering the modified domination Euler SomborBanhatti index, we introduce the modified domination Euler SomborBanhatti exponential of a graph G and defined it as

$${}^m DEU(G,x) = \sum_{uv\in E(G)} x^{\frac{1}{\sqrt{d_d(u)^2+d_d(v)^2+d_d(u)d_d(v)}}}.$$

Recently, some domination indices were studied in [124-138].

RESULTS FOR FRENCH WINDMILL GRAPHS

The French windmill graph F_n^m is the graph obtained by taking $m\geq 3$ copies of K_n, $n\geq 3$ with a vertex in common. The graph F_n^m is presented in Figure 10. The French windmill graph F_3^m is called a friendship graph.

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

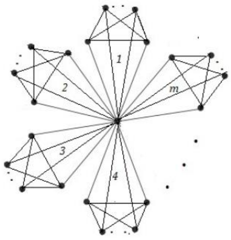

Figure 10. French windmill graph F_n^m

Let F be a French windmill graph F_n^m. Then

$d_d(u)=1$, if u is in center

$=(n-1)^{m-1}$, otherwise.

Theorem 13.1. Let F be a French windmill graph F_n^m. Then

$$DEU(F)=m(n-1)\sqrt{1+(n-1)^{(m-1)2}+(n-1)^{(m-1)}}$$
$$+[(mn(n-1)/2)-m(n-1)](n-1)^{(m-1)}\sqrt{3}.$$

Proof: In F, there are two sets of edges. Let E_1 be the set of all edges which are incident with the center vertex and E_2 be the set of all edges of the complete graph. Then

$$DEU(G)=\sum_{uv\in E(G)}\sqrt{d_d(u)^2+d_d(v)^2+d_d(u)d_d(v)}$$
$$=m(n-1)\sqrt{1^2+(n-1)^{(m-1)2}+1\times(n-1)^{(m-1)}}$$
$$+[(mn(n-1)/2)-m(n-1)]$$
$$\sqrt{(n-1)^{(m-1)2}+(n-1)^{(m-1)2}+(n-1)^{(m-1)}(n-1)^{(m-1)}}$$
$$=m(n-1)\sqrt{1+(n-1)^{(m-1)2}+(n-1)^{(m-1)}}$$
$$+[(mn(n-1)/2)-m(n-1)](n-1)^{(m-1)}\sqrt{3}.$$

Theorem 13.2. Let F be a French windmill graph F_n^m. Then

$$DEU(G,x)=m(n-1)x^{\sqrt{1+(n-1)^{(m-1)2}+(n-1)^{(m-1)}}}$$
$$+[(mn(n-1)/2)-m(n-1)]x^{\sqrt{3}(n-1)^{(m-1)}}.$$

Proof: From definition, we deduce

$$DEU(G,x)=\sum_{uv\in E(G)}x^{\sqrt{d_d(u)^2+d_d(v)^2+d_d(u)d_d(v)}}$$
$$=m(n-1)x^{\sqrt{1^2+(n-1)^{(m-1)2}+1\times(n-1)^{(m-1)}}}$$
$$+[(mn(n-1)/2)-m(n-1)]x^{\sqrt{(n-1)^{(m-1)2}+(n-1)^{(m-1)2}+(n-1)^{(m-1)}(n-1)^{(m-1)}}}$$
$$=m(n-1)x^{\sqrt{1+(n-1)^{(m-1)2}+(n-1)^{(m-1)}}}$$
$$+[(mn(n-1)/2)-m(n-1)]x^{\sqrt{3}(n-1)^{(m-1)}}.$$

Theorem 13.3. Let F be a French windmill graph F_n^m. Then

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

$$^{m}DEU(G)=\frac{m(n-1)}{\sqrt{1+(n-1)^{(m-1)2}+(n-1)^{(m-1)}}}+\frac{(mn(n-1)/2)-m(n-1)}{\sqrt{3}(n-1)^{(m-1)}}.$$

Proof: From definition, we deduce

$$^{m}DEU(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_d(u)^2+d_d(v)^2+d_d(u)d_d(v)}}$$

$$=\frac{m(n-1)}{\sqrt{1^2+(n-1)^{(m-1)2}+1\times(n-1)^{(m-1)}}}$$

$$+\frac{(mn(n-1)/2)-m(n-1)}{\sqrt{(n-1)^{(m-1)2}+(n-1)^{(m-1)2}+(n-1)^{(m-1)}(n-1)^{(m-1)}}}$$

$$=\frac{m(n-1)}{\sqrt{1+(n-1)^{(m-1)2}+(n-1)^{(m-1)}}}+\frac{(mn(n-1)/2)-m(n-1)}{\sqrt{3}(n-1)^{(m-1)}}.$$

Theorem 13.4. Let F be a French windmill graph F_n^m. Then

$$^{m}DEU(G,x)=m(n-1)x^{\frac{1}{\sqrt{1+(n-1)^{(m-1)2}+(n-1)^{(m-1)}}}}$$

$$+[(mn(n-1)/2)-m(n-1)]x^{\frac{1}{\sqrt{3}(n-1)^{(m-1)}}}.$$

Proof: From definition, we deduce

$$^{m}DEU(G,x)=\sum_{uv\in E(G)}x^{\frac{1}{\sqrt{d_d(u)^2+d_d(v)^2+d_d(u)d_d(v)}}}$$

$$=m(n-1)x^{\frac{1}{\sqrt{1^2+(n-1)^{(m-1)2}+1\times(n-1)^{(m-1)}}}}$$

$$+[(mn(n-1)/2)-m(n-1)]x^{\frac{1}{\sqrt{(n-1)^{(m-1)2}+(n-1)^{(m-1)2}+(n-1)^{(m-1)}(n-1)^{(m-1)}}}}$$

$$=m(n-1)x^{\frac{1}{\sqrt{1+(n-1)^{(m-1)2}+(n-1)^{(m-1)}}}}+[(mn(n-1)/2)-m(n-1)]x^{\frac{1}{\sqrt{3}(n-1)^{(m-1)}}}.$$

14. TEMPERATURE EULER SOMBOR BANHATTI INDEX

In [139], the temperature of a vertex u of a graph G is defined as

$$T(u)=\frac{d_u}{n-d_u}$$

where n is the number of vertices of G.
The above definition motivate us to introduce a new index, defined as

$$TEU(G)=\sum_{uv\in E(G)}\sqrt{T(u)^2+T(v)^2+T(u)T(v)}$$

which we propose to be named as temperature Euler Sombor Banhatti index.
Considering the domination Euler Sombor Banhatti index, we introduce the domination Euler Sombor Banhatti exponential of a graph G and defined it as

$$TEU(G,x)=\sum_{uv\in E(G)}x^{\sqrt{T(u)^2+T(v)^2+T(u)T(v)}}.$$

We define the modified domination Euler Sombor Banhatti index of a graph G as

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

$${}^{m}TEU(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{T(u)^2+T(v)^2+T(u)T(v)}}.$$

Considering the modified domination Euler SomborBanhatti index, we introduce the modified domination Euler SomborBanhatti exponential of a graph G and defined it as

$${}^{m}TEU(G,x)=\sum_{uv\in E(G)}x^{\frac{1}{\sqrt{T(u)^2+T(v)^2+T(u)T(v)}}}.$$

Recently, some temperature indices were studied in [139-150].

RESULTS FOR r-REGULAR GRAPHS

Let G be an r-regular graph with n vertices, $r\geq 2$ and $\frac{nr}{2}$ edges. Then $T(u)=\frac{r}{n-r}$, for any vertex u in G.

Theorem 14.1. Let G be an r-regular graph with n vertices and $r\geq 2$. Then

$$TEU(G)=\frac{\sqrt{3}nr^2}{2(n-r)}.$$

Proof: We deduce

$$TEU(G)=\sum_{uv\in E(G)}\sqrt{T(u)^2+T(v)^2+T(u)T(v)}$$

$$=\frac{nr}{2}\sqrt{\left(\frac{r}{n-r}\right)^2+\left(\frac{r}{n-r}\right)^2+\left(\frac{r}{n-r}\right)\left(\frac{r}{n-r}\right)}$$

$$=\frac{nr}{2}\frac{\sqrt{3}r}{n-r}=\frac{\sqrt{3}nr^2}{2(n-r)}.$$

Theorem 14.2. Let G be an r-regular graph with n vertices and $r\geq 2$. Then

$$TEU(G,x)=\frac{nr}{2}x^{\frac{\sqrt{3}r}{n-r}}.$$

Proof: From definition, we deduce

$$TEU(G,x)=\sum_{uv\in E(G)}x^{\sqrt{T(u)^2+T(v)^2+T(u)T(v)}}$$

$$=\frac{nr}{2}x^{\sqrt{\left(\frac{r}{n-r}\right)^2+\left(\frac{r}{n-r}\right)^2+\left(\frac{r}{n-r}\right)\left(\frac{r}{n-r}\right)}}$$

$$=\frac{nr}{2}x^{\frac{\sqrt{3}r}{n-r}}.$$

Theorem 14.3. Let G be an r-regular graph with n vertices and $r\geq 2$. Then

$${}^{m}TEU(G)=\frac{n(n-r)}{2\sqrt{3}}.$$

Proof: From definition, we deduce

$${}^{m}TEU(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{T(u)^2+T(v)^2+T(u)T(v)}}$$

$$=\frac{nr}{2\sqrt{\left(\frac{r}{n-r}\right)^2+\left(\frac{r}{n-r}\right)^2+\left(\frac{r}{n-r}\right)\left(\frac{r}{n-r}\right)}}$$

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

$$=\frac{nr(n-r)}{2\sqrt{3}r}=\frac{n(n-r)}{2\sqrt{3}}.$$

Theorem 14.4. Let G be an r-regular graph with n vertices and $r \geq 2$. Then

$${}^{m}TEU(G,x)=\frac{nr}{2}x^{\frac{(n-r)}{\sqrt{3}r}}.$$

Proof: From definition, we deduce

$${}^{m}TEU(G,x)=\sum_{uv\in E(G)} x^{\frac{1}{\sqrt{T(u)^2+T(v)^2+T(u)T(v)}}}$$

$$=\frac{nr}{2}x^{\frac{1}{\sqrt{\left(\frac{r}{n-r}\right)^2+\left(\frac{r}{n-r}\right)^2+\left(\frac{r}{n-r}\right)\left(\frac{r}{n-r}\right)}}}$$

$$=\frac{nr}{2}x^{\frac{(n-r)}{\sqrt{3}r}}.$$

15. EULER SOMBOR E-BANHATTI INDEX

If $e=uv$ is an edge of G, then the vertex u and edge e are incident as are v and e. Let $d_G(e)$ denote Recall the degree of an edge e in G, which is defined by $d_G(e) = d_u + d_v - 2$ with $e=uv$.

In [151], Kulli defined the Banhatti degree of a vertex u of a graph G as

$$B(u)=\frac{d_G(e)}{n-d_u}$$

where n is the number of vertices of G and the vertex u and edge e are incident in G.

The above definition motivates us to introduce a new index, defined as

$$EBEU(G)=\sum_{uv\in E(G)}\sqrt{B(u)^2+B(v)^2+B(u)B(v)}$$

which we propose to be named as Euler Sombor E-Banhatti index.

Considering the Euler Sombor E-Banhatti index, we introduce the Euler Sombor E-Banhatti exponential of a graph G and defined it as

$$EBEU(G,x)=\sum_{uv\in E(G)} x^{\sqrt{B(u)^2+B(v)^2+B(u)B(v)}}.$$

We define the modified Euler Sombor E-Banhatti index of a graph G as

$${}^{m}EBEU(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{B(u)^2+B(v)^2+B(u)B(v)}}.$$

Considering the modified Euler Sombor E-Banhatti index, we introduce the modified Euler Sombor E-Banhatti exponential of a graph G and defined it as

$${}^{m}EBEU(G,x)=\sum_{uv\in E(G)} x^{\frac{1}{\sqrt{B(u)^2+B(v)^2+B(u)B(v)}}}.$$

Recently, some E-Banhatti indices were studied in [151-157].

RESULTS FOR r-REGULAR GRAPHS

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

Let G be an r-regular graph with n vertices and $r \geq 2$ and $\frac{nr}{2}$ edges. For any edge $uv=e$ in G, $d_G(e)=2r-2$. Then

$$B(u)=\frac{2r-2}{n-r}, \quad \text{for any vertex } u \text{ in } G.$$

Theorem 15.1. Let G be an r-regular graph with n vertices and $r \geq 2$. Then

$$EBEU(G)=\frac{\sqrt{3}nr(r-1)}{(n-r)}.$$

Proof: From definition, we deduce

$$EBEU(G)=\sum_{uv\in E(G)}\sqrt{B(u)^2+B(v)^2+B(u)B(v)}$$

$$=\frac{nr}{2}\sqrt{\left(\frac{2r-2}{n-r}\right)^2+\left(\frac{2r-2}{n-r}\right)^2+\left(\frac{2r-2}{n-r}\right)\left(\frac{2r-2}{n-r}\right)}$$

$$=\frac{nr}{2}\frac{\sqrt{3}(2r-2)}{(n-r)}=\frac{\sqrt{3}nr(r-1)}{(n-r)}.$$

Theorem 15.2. Let G be an r-regular graph with n vertices and $r \geq 2$. Then

$$EBEU(G,x)=\frac{nr}{2}x^{\frac{\sqrt{3}(2r-2)}{n-r}}.$$

Proof: From definition, we deduce

$$EBEU(G,x)=\sum_{uv\in E(G)}x^{\sqrt{B(u)^2+B(v)^2+B(u)B(v)}}$$

$$=\frac{nr}{2}x^{\sqrt{\left(\frac{2r-2}{n-r}\right)^2+\left(\frac{2r-2}{n-r}\right)^2+\left(\frac{2r-2}{n-r}\right)\left(\frac{2r-2}{n-r}\right)}}$$

$$=\frac{nr}{2}x^{\frac{\sqrt{3}(2r-2)}{n-r}}.$$

Theorem 15.3. Let G be an r-regular graph with n vertices and $r \geq 2$. Then

$$^{m}EBEU(G)=\frac{nr(n-r)}{2\sqrt{3}(2r-2)}.$$

Proof: From definition, we deduce

$$^{m}EBEU(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{B(u)^2+B(v)^2+B(u)B(v)}}$$

$$=\frac{nr}{2\sqrt{\left(\frac{2r-2}{n-r}\right)^2+\left(\frac{2r-2}{n-r}\right)^2+\left(\frac{2r-2}{n-r}\right)\left(\frac{2r-2}{n-r}\right)}}$$

$$=\frac{nr(n-r)}{2\sqrt{3}(2r-2)}.$$

Theorem 15.4. Let G be an r-regular graph with n vertices and $r \geq 2$. Then

$$^{m}EBEU(G,x)=\frac{nr}{2}x^{\frac{(n-r)}{\sqrt{3}(2r-2)}}.$$

Proof: From definition, we obtain

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

$$ {}^{m}EBEU(G,x)=\sum_{uv\in E(G)} x^{\frac{1}{\sqrt{B(u)^2+B(v)^2+B(u)B(v)}}} $$

$$ =\frac{nr}{2}x^{\frac{1}{\sqrt{\left(\frac{2r-2}{n-r}\right)^2+\left(\frac{2r-2}{n-r}\right)^2+\left(\frac{2r-2}{n-r}\right)\left(\frac{2r-2}{n-r}\right)}}} $$

$$ =\frac{nr}{2}x^{\frac{(n-r)}{\sqrt{3}(2r-2)}}. $$

16. CONCLUSION

In this paper, we have defined some Euler Sombor Banhatti indices and modified Euler Sombor Banhatti indices and their corresponding exponentials of a graph. Also, these newly defined Euler Sombor Banhatti indices and modified Euler Sombor Banhatti indices of certain graphs are determined.

REFERENCES

1. V.R.Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India (2012).
2. V.R.Kulli, Graph indices, in *Hand Book of Research on Advanced Applications of Application Graph Theory in Modern Society*, M. Pal. S. Samanta and A. Pal, (eds.) IGI Global, USA (2019) 66-91.
3. I.Gutman, Relating Sombor and Euler indices, *Vojnotehnickiglasnik*, 72(1) (2024). DOI:10.5937/ vojtehg72-48818.
4. V.R.Kulli, Nirmala alpha Gourava and modified Nirmala alpha Gourava indices of some dendrimers, *International Journal of Mathematics and Computer Research*, 12(5) (2024).
5. I.Gutman, B.Furtula and M.S.Oz, Geometric approach to vertex degree based topological indices-Elliptic Sombor index theory and application, *International Journal of Quantum Chemistry*, to appear.
6. V.R.Kulli, Modified elliptic Sombor index and its exponential of a graph, *International Journal of Mathematics and Computer Research*, 12(1) (2024) 3949-3954.
7. V.R.Kulli, Multiplicative elliptic Sombor and multiplicative modified elliptic Sombor indices, *Annals of Pure and Applied Mathematics*, 29(1) (2024) 19-23.
8. V.R.Kulli, Reverse elliptic Sombor and modified reverse elliptic Sombor indices, *International Journal of Mathematical Archive*, 15(1) (2024) 1-7.
9. V.R.Kulli, Reduced elliptic and modified reduced elliptic indices of some chemical structures, *International Journal of Engineering Sciences & Research Technology*, 13(2) (2024) 11-17.
10. V.R.Kulli, Elliptic Revan index and its exponential of certain networks, *International Journal of Mathematics and Computer Research*, 12(2) (2024) 4055-4061.
11. V.R.Kulli, Modified elliptic Revan index of two families of nanotubes, *Annals of Pure and Applied Mathematics*, 29(2) (2024) 103-107.
12. S.Alikhani and N.Ghanbari, Sombor index of polymers, *MATCH Commun. Math. Comput. Chem.* 86 (2021).
13. R.Aguilar-Sanchez, J.A.Mendez-Bermudez and L.MSigaarreta, Normalized Sombor indices as complexity measures of random graphs, arXiv (2021) DOI: arXiv 2106.o3190.
14. R.Cruz, I.Gutman and J.Rada, Sombor index of chemical graphs, *Appl. Math. Comput.* 399 (2021) 126018.
15. H.Deng, Z.Tang and R.Wu, Molecular trees with extremal values of Sombor indices, *Int. J. Quantum Chem.*DOI: 10.1002/qua.26622.
16. I.Gutman, Sombor indices-back to geometry, *Open Journal of Discrete Applied Matematics*,5(2) (2022) 1-5.
17. I.Gutman, V.R.Kulli and I.Redzepovic, Irregularity Sombor index, *Bull. Acad. Serbe Sci. Math. Arts (Cl.Sci. Math. Natur.)* 156 (2023) 31-37.
18. B.Horoldagva and C.Xu, On Sombor index of graphs, *MATCH Commun. Math. Comput. Chem.* 86 (2021).
19. V.R.Kulli, δ-Sombor index and its exponential of certain nanotubes, *Annals of Pure and Applied Mathematics*, 23(1) (2021) 37-42.

20. V.R.Kulli, Delta F, modified delta Sombor and delta sigma indices of certain nanotubes and networks, *International Journal of Mathematics and Computer Research*, 11(10) (2023) 3813-3820.
21. V.R.Kulli, Multiplicative *KG*Sombor indices of some networks, *International Journal of Mathematics Trends and Technology*, 68(10) (2022) 1-7.
22. Z.Lil, T.Zhou, V.R.Kulli and L.Miao, On the first Banhatti-Sombor index, *J. Int. Math. Virt. Inst.* 11 (2021) 53-68.
23. B. Basavanagoud, A. P. Barangi and S. M. Hosamani, First neighbourhood Zagreb index of some nanostructures, *Proceedings IAM.* 7(2) (2018) 178-193.
24. A.Graovac, M. Ghorbani and M.A. Hosseinzadeh, Computing fifth geometric-arithmetic index of nanostar dendrimers, *Journal of Mathematical Nanoscience*, 1(1) (2011) 33-42.
25. V.R.Kulli, Neighborhood Dakshayani indices, *International Journal of Mathematical Archive*, 10(7) (2019) 23-31.
26. V.R.Kulli, Neighborhood indices of nanostructures, *International Journal of Current Research in Science and Technology*, 5(3) (2019) 1-14.
27. V.R.Kulli, Multiplicative neighborhood indices, *Annals of Pure and Applied Mathematics*, 19(2) (2019) 175-181.
28. V.R.Kulli, General fifth *M*-Zagreb indices and fifth *M*-Zagreb polynomials of PAMAM dendrimers, *International Journal of Fuzzy Mathematical Archive*, 13(1) (2017) 99-103.
29. V.R.Kulli, Computing fifth arithmetic-geometric index of certain nanostructures, *Journal of Computer and Mathematical Sciences*, 8(5) (2017) 196-201.
30. V.R.Kulli, Some new fifth multiplicative Zagreb indices of PAMAM dendrimers, *Journal of Global Research in Mathematics*, 5(2) (2018) 82-86.
31. S. Mondal, N. De, A. Pal, On neighbourhood index of product of graphs, arXivi1805.05273v1 [math.Co] 14 May 2018.
32. S.Mondal, M.A. Ali, N.De and A. Pal, Bounds for neighborhood Zagreb index and its explicit expressions under some graph operations, *Proyecciones*, 39(4) (2020) 799-819.
33. V.R. Kulli, On *KV* indices and their polynomials of two families of dendrimers, *International Journal of Current Research in Life Sciences*,7(9) (2018) 2739-2744.
34. V.R. Kulli, On hyper-*KV* and square *KV* indices and their polynomials of certain families of dendrimers, *Journal of Computer and Mathematical Sciences*, 10(2) (2019) 279-286.
35. V.R. Kulli, On connectivity *KV* indices of certain families of dendrimers, *International Journal of Mathematical Archive*, 10(2) (2019) 14-17.
36. V.R. Kulli, Multiplicative connectivity *KV* indices of dendrimers, *Journal of Mathematics and Informatics*, 15(1) (2019) 1-7.
37. V.R.Kulli, δ-Sombor index and its exponential of certain nanotubes, *Annals of Pure and Applied Mathematics*, 23(1) (2021) 37-42.
38. V.R.Kulli, δ-Banhatti, hyper δ-Banhatti indices and their polynomials of certain networks, *International Journal of Engineering Sciences and Research Technology*, 10(3) (2021) 1-8.
39. S.B.B.Altindag, I.Milovannovic and E.MilovanocicM.Matejic, S.Stankov , Remark on delta and reverse degree indices, Sci. Pub. State Univ. Novi Pazar, Ser. A: Appl. Math. Inform. Mech. 15(1) (2022) 37-47.
40. V.R.Kulli, Delta F, modified delta Sombor and delta sigma indices of certain nanotubes and networks, *International Journal of Mathematics and Computer Research*, 11(10) (2023) 3813-3820.
41. V.R.Kulli, Delta Banhatti-Sombor indices of certain networks, *International Journal of Mathematics and Computer Research*, 11(11) (2023) 3875-3881.
42. V.R.Kulli, Delta atom bond connectivity indices of certain networks, *International Journal of Mathematics and its Applications*, 11(4) (2023) 89-98.
43. V.R.Kulli, On delta harmonic and multiplicative delta harmonic indices of some nanostructures, *International Journal of Mathematics and Statistics Invention*, 11(5) (2023) 38-45.
44. B.K.Majhi, V.R.Kulli and I.N.Cangul, Sum connectivity delta Banhatti and product connectivity delta Banhatti indices of certain nanotubes, *Montes Taurus J.Pure Appl. Math.* 5(2) (2023) 89-97.
45. E.Milovannovic and I.Milovanocic, M.Matejic, S.Stankov and S.B.B.Altindag, Bounds for the reverse (delta) first Zagreb indices, Sci. Pub. State Univ. Novi Pazar, Ser. A: Appl. Math. Inform. Mech. 15(1) (2023) 49-59.

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

46. A.Ashraf and S.Akhter, Revan indices and Revan polynomials of silicon carbide graphs, *International Journal of Engineering Research& Technology*, 8(9) (2019) 353-361.
47. A.Q.Baig, M.Naeem and W.Gao, Revan and Hyper-Revan indices of octahedral and icosahedral networks, *Applied Mathematics and Nonlinear Sciences*, 3(1) (2018) 33-40.
48. R.Aguilar-Sanchez, I.F.Herrera-Gonzalez, J.A.Mendez-Bermudez and J.M.Sigaareta, Revan degree indices on random graphs, arXiv: 2210.04749v1 [math. CO] 10 Oct. 2022.
49. M.Kamran, S.Delen, R.H.Khan, N.Salamat, A.Q.Baig, I.N.Cangul and Md.A.alam, Physico-Chemical Characterization of amylase and amylopectin using Revan topological indices, *Journal of Mathematics*, vol.2022, 12 pages, Article ID 2840217, 2022.
50. M.Kamran, N.Salamat, R.H.Khan, M.A.Ullah, M.S.Hameed and M.K.Pandit, Computation of Revan topological indices for phenol-formaldehyde resin, *Journal of Mathematics*, vol.2022, 10 pages, Article ID 8548771, 2022.
51. V.R. Kulli, Revan indices of oxide and honeycomb networks, *International Journal of Mathematics and its Applications*, 5(4-E), (2017) 663-667.
52. V.R.Kulli, Multiplicative Revan and multiplicative hyper Revan indices of certain networks, *Journal of Computer and Mathematical Sciences*, 8(12), (2017) 750-757.
53. V.R. Kulli, Hyper-Revan indices and their polynomials of silicate networks, *International Journal of Current Research in Science and Technology*, 4(3) (2018) 17-21.
54. V.R.Kulli and I.Gutman, Revan Sombor index, *Journal of Mathematics and Informatics*,22(2022)23-27. DOI: http://dx.doi.org/10.22457/jmi.v22a03205.
55. V.R.Kulli, J.A.Mendez-Bermudez, J.M.Rodriguez and J.M.Sigarreta, Revan Sombor indices: Analytical and statistical study, on random graphs, *Mathematical Boisciences and Engineering*, 20(2) (2022) 1801-1819. DOI: 10.3934/mbe.2023082.
56. A.Mahboob, G.Muhiuddin, I.Siddique and S.M.Alam, A view of Banhatti and Revan indices in chemical graphs, *Journal of Mathematics*, vol.2022, 8 pages, Article ID 5680712, 2022.
57. D.Narasmhan, R.Vignesh and K.Desikan, Results on Revan and hyper Revan indices of some HEX derived networks, *Fuzzy Mathematical Analysis and Advances in Computational Mathematics*, Springer Singapore (2022) 209-220.
58. M.Priyadharshini, P.Kandan, E.Chandrasekaran and A.J.Kennedy, Revan weighted PI index on some product of graphs, *TWMS J. APP. And Eng. Math.* 11 Special issue (2021) 203-215.
59. D.Zhao, M.A.Zahid, R.Irfan et al., Banhatti, Revan and hyper indices of silicon carbide Si2C3-III[n, m], *Open Chemistry*, 19(1) (2021) 646-652.
60. S.Ediz Maximal graphs of the first reverse Zagreb beta index, *TWMS J. Appl. Eng. Math.* 8(1a) (2018) 306-310.
61. S.Ediz and M.Cancan, Reverse Zagreb indices of certain product of graphs, *International Journal of Mathematics and Computer Science*, 11(1) (2016) 51-58.
62. W.Gao, M.Younas, A.Farooq, A.R.Virk and W.Nazeer, Some reverse degree based topological indices and polynomials of dendrimers, *Methematics*, 6 (2018) 214.
63. V.R. Kulli, On the product of connectivity reverse index of silicate and hexagonal networks, *International Journal of Mathematics and its Applications*, 5(4-B), (2017) 175-179.
64. V.R.Kulli, Geometric-arithmetic reverse and sum connectivity reverse indices of silicate and hexagonal networks, *International Journal of Current Research in Science and Technology*, 3(10), (2017) 29-33.
65. V.R. Kulli, Reverse Zagreb and reverse hyper Zagreb indices and their polynomials of rhombus silicate networks, *Annals of Pure and Applied Mathematics*, 16(1) (2018) 47-51.
66. V.R. Kulli, Atom bond connectivity reverse and product connectivity reverse indices of oxide and honeycomb networks, *International Journal of Fuzzy Mathematical Archive*, 15(1) (2018) 1-5. DOI: http://dx.doi.org/10.22457/ijfma.v15n1a1.
67. V.R.Kulli, On augmented reverse index and its polynomial of certain nanostar dendrimers, *International Journal of Engineering Sciences and Research Technology*, 7(8) (2018) 237-243.
68. V.R.Kulli, Computing two arithmetic-geometric reverse indices of certain networks, *International Research Journal of Pure Algebra*, 8(8) (2018) 43-49.
69. V.R.Kulli, Reverse Nirmala index, *International Journal of Engineering Sciences & Research Technology*, 11(8) (2022) 12-19. DOI: 10.5281/zenodo.7079293.
70. K.G.Mirajkar and B.Pooja, Degree based reverse connectivity indices of dendrimers, *RESEARCH REVIEW International Journal of Multidisciplinary*, 4(5) (2019) 506-513.

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

71. A.R.Virk, M.N.Jhangeer and M.A.Rehman, Reverse Zagreb and Reverse hyper Zagreb indices for silicon carbide Si2C3I[r, s] and Si2C3II[r, s], *Eng, Appl. Sci. Lett.*, 1(2) (2018) 37-50.
72. F.Asif, Z.Zahid and M.Cancan, On some leap indices of polycyclic aromatic hydrocarbons, *Journal of Discrete Mathematical Sciences and Cryptography*, 23(6) (2020) 1269-1277.
73. F.Asif, Z.Zahid and S.Zafar, Leap Zagreb and leap hyper Zagreb indices of Jahangir and Jahangir derived graphs, *Engineering and Applied Science Letters*, 3(2) (2020) 1-8.
74. A.Alsinal, A.Alwardi and N.D.Soner, Topological Properties of graphene using φk-polynomial, *Proceedings of the Jangjeon Mathematical Society*, 24(3) (2021) 375-388.
75. B.Basavagoud and E.Chitra, On leap hyper Zagreb indices of some nanostructures, *International Journal Mathematics Trends and Technology*, 64(1) (2018) 30-36.
76. F.Dayan, M.Javaid and M.A.Rehman, On leap Gourava indices of some wheel related graphs, *Scientific Inquiry and Review*, 2(4) (2018) 14-24.
77. I.Gutman, Z.Shao, Z.Li, S.Wang and P.Wu, Leap Zagreb indices of trees and unicyclic graphs, 3(2) (2018) 179-194, DOI:10.22049/CCO.2018.26285.1092.
78. V.R. Kulli, Leap Onhyper Zagreb indices and their polynomials of certain graphs, *International Journal of Current Research in Life Sciences*, 7(10) (2018) 2783-2791.
79. V.R. Kulli, Minus leap and square leap indices and their polynomials of some special graphs, *International Research Journal of Pure Algebra*, 8(11) (2018) 54-60.
80. V.R. Kulli, Product connectivity leap index and ABC leap index of helm graphs, *Annals of Pure and Applied Mathematics*, 18(2) (2018) 189-193.
81. V.R. Kulli, Sum connectivity leap index and geometric-arithmetic leap index of certain windmill graphs, *Journal of Global Research in Mathematical Archives*, 6(1) (2019) 15-20.
82. V.R. Kulli, On F-leap indices and F-leap polynomials some graphs, *International Journal of Mathematical Archive*, 9(12) (2018) 41-49.
83. V.R. Kulli, Inverse sum leap index and harmonic leap index of certain windmill graphs, *International Research Journal of Pure Algebra*, 9(2) (2019) 11-17.
84. V.R. Kulli, P.Jakkannavar and B.Basavagoud, Computation of leap hyper Zagreb indices of certain windmill graphs, *International Journal of Scientific Research im Mathematical and Statistical Sciences*, 6(2) (2019) 75-79.
85. B.K.Majhi, V.R.Kulli and I.N.Cangul, Nirmala leap indices of some chemical drugs against COVID-19, *Journal of Discrete Mathematics and its Applications*, 10(1) (2023) 13-21.
86. A.M.Naji, N.D.Soner and I.Gutman, On leap Zagreb indices of graphs, *Communications in Combinatorics and Optimization*, 2(2) (2017) 99-117.
87. F.Harary, Status and contrastatus, *Sociometry*, 22 (1959) 23-43.
88. C.Adiga and R.Malpashree, The degree status connectivity index of graphs and its multiplication version, *South Asian Journal of Mathematics*, 6(6) (2016) 288-299.
89. S.A.Kauser and A.Khan, A study of some new multiplicative status indices of some special graphs, *International Journal of Grid and Distributed Computing*,13(20 (2020) 1306-1313.
90. V.R.Kulli, Some new status indices of graphs, *International Journal of Mathematics Trends and Technology*, 65(10) (2019) 70-76.
91. V.R.Kulli, Computation of some new status neighborhood indices of graphs, *International Research Journal of Pure Algebra*, 10(6) (2020) 6-13.
92. V.R.Kulli, Multiplicative status neighborhood indices of graphs, *International Journal of Engineering Sciences & Research Technology*, 9(6) (2020) 110-120.
93. V.R.Kulli, Computation of square status and inverse sum status indices of certain graphs, *Annals of Pure and Applied Mathematics*, 22(1) (2020) 1-6. DOI: http://dx.doi.org/10.22457/apam.v22n1a01675.
94. V.R.Kulli, Some new status neighborhood indices of graphs, *International Journal of Mathematics Trends and Technology*, 66(9) (2020) 139-153.
95. V.R.Kulli, Inverse sum status and modified status indices of graphs and their multiplicative versions, *International Journal of Recent Scientific Research*, 11, 10(A) (2020) 39753-39759. DOI:http://dx.doi.org/10.24327/ijrsr.2020.1110.5559.
96. V.R.Kulli, Multiplicative connectivity status neighborhood indices of graphs, *International Journal of Engineering Sciences & Research Technology*, 9(12) (2020) 59-68. http://doi.org/10.29121/ijesrt.v9.i12.2020.8.

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

97. V.R.Kulli, The (*a*, *b*)-status neighborhood Dakshayani index, *International Journal of Mathematics Trends and Technology*, 67(1) (2021) 79-87. https://doi.org/ 10.14445/22315373/IJMTT-V67I1P512.
98. H.S.Ramane and A.S.Yalnaik, Status connectivity indices of graphs and its applications to thr boiling point of benzenoid hydrocarbons, *J. Appl. Math. Comput.* DOI: 10.1007/s12190-016-1052-5 (2016).
99. D.S.Revankar, P.S.Hande and S.P.Hande, On atom bond connectivity status index of graphs, *Advances in Mathematics: Scientific Journal*, 10(3) (2021) 1197-1213.
100. K.P.Narayankar and D.Selvan, Geometric-arithmetic status index of graphs, *International Journal of Mathematical Archive*, 8(7) (2017) 230-233.
101. K.R.Sridhar and M.B.Kattimani, The (a, b)-status indices of central graphs of some standard graph, *International Journal of Basic Sciences and Applied Computing*, 8(8) (2022) 12-17
102. M.Chellali, T.W.Haynes, T.Hedetniemi and T.M.Lewis, On ve-degrees and ev-degrees in graphs, *Discrete Mathematics*, 340(2) (2017) 31-38.
103. S.Ediz, Predicting some physicochemical properties of octane isomers: A topological approach using ev-degree and ve=degree Zagreb indices, *International Journal of Systems Science and Applied Mathematics*, 2(5) (2017) 87-92.
104. S.Ediz, On ve-degree molecular topological properties of silicate and oxygen networks, *International Journal of Computing Science and Mathematics*, 9(1) (2018) 1-12.
105. M.Cancan and M.S.Aldemir, On ve-degree and ev-degree Zagreb indices of titania nanotubes, *American Journal of Chemical Engineering*, 5(6) (2017) 163-168.
106. V.R. Kulli, Two new arithmetic-geometric ve-degree indices, *Annals of Pure and Applied Mathematics*, 17(1) (2018) 107-112. DOI: http://dx.doi.org/10.22457/apam.v17n1a12.
107. V.R.Kulli, General multiplicative ve-degree indices of dominating oxide and regular triangulate oxide networks, *Journal of Global Research in Mathematical Archives*, 5(5) (2018) 60-64.
108. V.R.Kulli, Multiplicative connectivity ve-degree indices of dominating oxide and regular triangulate oxide networks, *International Journal of Current Advanced Research*, 7, 4(J) (2018) 11961-11964.
109. V.R. Kulli, On ve-degree indices and their polynomials of dominating oxide networks, *Annals of Pure and Applied Mathematics*, 18(1) (2018) 1-7. DOI: http://dx.doi.org/10.22457/apam.v18n1a1.
110. V.R.Kulli, Computing the F-ve-degree index and its polynomial of dominating oxide and regular triangulate oxide networks, *International Journal of Fuzzy Mathematical Archive*, 16(1) (2018) 1-6. DOI: http://dx.doi.org/10.22457/ijfma.v16n1a1.
111. V.R.Kulli, Computing ev-degree and multiplicative ev-degree indices of certain chemical structures, *International Journal of Engineering Sciences & Research Technology*, 9(7) (2020) 54-65. https://doi.org/10.29121/ijesrt.v9.i7.2020.7.
112. S.A.K.Kirmani, P.Ali, F.Azam and P.A.Alvi, On ve-degree and ev-degree topological properties of hyaluronic acid-anticancer drug conjugate with QSPR, *Journal of Chemistry*, vol. 2021, Article ID 3860856, 23 pages.
113. B.Sahin and S.Ediz, On ev-degree and ve-degree topological indices, *Iranian J. Math, Chem.*, 9(4) (2018) 263-277.
114. B.Basavangoud and P.Jakkannavar, Kulli-Basava indices of graphs, *J. Appl. Engg. Research*, 14(1) (2018) 325-347.
115. V.R. Kulli, New connectivity topological indices, *Annals of Pure and Applied Mathematics*, 20(1) (2019) 1-8. DOI: http://dx.doi.org/10.22457/apam.622v20n1a1.
116. V.R. Kulli Some new Kulli-Basava topological indices, *EarthlineJournal of Mathematical Sciences*, 2(2) (2019) 343-354
117. V.R. Kulli New Kulli-Basava indices of graphs, *International Research Journal of Pure Algebra*, 9(7) (2019) 58-63.
118. V.R. Kulli, Multiplicative Kulli-Basava and multiplicative hyper Kulli-Basava indices of some graphs, *International Journal of Mathematical Archive*, 10(8) (2019) 18-24.
119. V.R.Kulli, Some new multiplicative connectivity Kulli-Basava indices, *International Journal of Mathematics Trends and Technology*, 65(9) (2019) 18-23.
120. V.R.Kulli, The multiplicative (a, b)-Kulli-Basava indices of graphs, *International Journal of Mathematical Archive*, 11(11) (2020) 9-15.
121. S.Afridi, M.Y.Khan and G.Ali, On generalized topological indices for some special graphs, (2022).
122. A.S.Nair, Some new topological indices of friendship graph, *International Journal for Research*, 11(10) (2023) 154-158.

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

123. M.Sharjeel, M.M.Abbas, M.R.Farahani, M.Alaeiyan and M.Cancam, On new connectivity topological indices of certain molecular graphs, *Eur, Chem. Bull.*, 12(Special issue 1) (2023) 15-21.
124. A.M.H.Ahmed, A.Alwardi and M.RubySalestina, On domination topological indices of graphs, *International Journal of Analysis and Applications,* 19(1) (2021) 47-64.
125. V.R.Kulli, Domination Nirmala indices of graphs, *International Journal of Mathematics and Computer Research,* 11(6) (2023) 3497--3502. DOI: https://doi.org/10.47191/ijmcr/v11i6.05.
126. V.R.Kulli, Domination augmented Banhatti, domination augmented Banhatti sum indices of certain chemical drugs, *International Journal of Mathematics and Computer Research,* 11(7) (2023) 3558--3564. DOI: https://doi.org/10.47191/ijmcr/v11i7.10.
127. V.R.Kulli, Modified domination Sombor index and its exponential of a graph, *International Journal of Mathematics and Computer Research,* 11(8) (2023) 3639-3644. DOI:10.47191/ijmcr/v11i8.03.
128. V.R.Kulli, Irregularity domination Nirmala and domination Sombor indices of certain drugs, *International Journal of Mathematical Archive,* 14(8) (2023) 1-7.
129. V.R.Kulli, Gourava domination indices of graphs, *International Journal of Mathematics and Computer Research,* 11(8) (2023) 3680-3684. DOI:10.47191/ijmcr/v11i8.08.
130. V.R.Kulli, Sum and product connectivity Gourava domination indices of graphs, *International Journal of Mathematics and Statistics Invention,* 11(4) (2023) 35-43. DOI:10.35629/4767-11043543.
131. V.R.Kulli, Modified domination and domination Banhatti indices of some chemical drugs, *International Journal of Mathematics Trends and Technology,* 69(9) (2023) 1-7. https://doi.org/10.14445/22315373/IJMTT-V69I9P501.
132. V.R.Kulli, On hyper Gourava domination indices, *International Journal of Engineering Sciences & Research Technology,* 12(10) (2023) 12-20. DOI: 10.5281/zenodo.10036777.
133. V.R.Kulli, Domination atom bond sum connectivity indices of certain nanostructures, *International Journal of Engineering Sciences & Research Technology,* 12(11) (2023) 10-17. DOI: 10.5281/zenodo.10209674.
134. H.Ahmed, A.Alwardi, M.RubySalestina and N.D.Soner, Domination, γ-domination topological indices and φ-polynomial of some chemical structures applied for the treatment of COVID-19 patients, *Biointerface Research in Applied Chemistry,* 11(5) (2021) 13290-13302.
135. S.Raju, Puttuswamy and A.Alsinai, Different type's domination topological indices of some chemical drugs graph, *Eur. Chem. Bull.* 12(5) (2023) 4334-4345.
136. H.Ahmed, M.RubySalestina, A.Alwardi and N.D.Soner, "Forgotten domination, hyper domination and modified forgotten domination indices of graphs," *Journal of Discrete Mathematical Sciences and Cryptography*, vol. 24, no. 2, pp 553-568, 2021.
137. S.Raju, Puttuswamy and S.R.Nayaka, On the second domination hyper index of graph and some graph operations, *Advances and Applications in Discrete Mathematics*, 39(1) (2023) 125-143.
138. A.A.Shashidhar, H.Ahmed, N.D.Soner and M.Cancan, Domination version: Sombor index of graphs and its significance in predicting physicochemical properties of butane derivatives, *Eurasian Chemical Communications*, 5 (2023) 91-102.
139. S.Fajtlowicz, On conjectures of graffiti, *Annals of Discrete Mathematics,* 72 (1988) 113-118.
140. V.R.Kulli, Multiplicative (a, b)-KA temperature indices of certain nanostructures, *International Journal of Mathematics Trends and Technology,* 66(5) (2020) 137-142. Doi:10.14445/22315373/IJMTT-V66I5P518.
141. V.R.Kulli, Computing some multiplicative temperature indics of certain networks, *Journal of Mathematics and Informatics* 18 (2020) 139-143. DOI: http://dx.doi.org/10.22457/jmi.av18a12171.
142. V.R.Kulli, Inverse sum temperature index and multiplicative inverse sum temperature index of certain nanotubes, *International Journal of Recent Scientific Research*, 12, 01(c) (2021) 40635-40639. DOI: http://dx.doi.org/10.24327/ijrsr.2021.1201.5734.
143. V.R.Kulli, The (*a, b*)-*KA* temperature indices of tetrameric 1,3-adamantane, *International Journal of Recent Scientific Research,* 12, 02(c) (2021) 40929-40933. DOI: http://dx.doi.org/10.24327/ijrsr.2021.1202.5793.
144. V.R.Kulli, Temperature Sombor and temperature Nirmala indices, *International Journal of Mathematics and Computer Research,* 10(9) (2022) 2910-2915. DOI: https://doi.org/10.47191/ijmcr/v10i9.04.
145. I.Gutman, Note on the temperature Sombor index, *VijnotehanckiGlasnik/ Military Technical courier,* 71(3) (2023) 507-515.

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

146. S.Iran, M.S.R.Chowdhury, A.R.Virk and M.Cancan, Temperature based invariants for bridge graphs, *Jilin DaxueXuebao (Gongxueban) Journal of Jilin University (Engineering and Technology Edition)* 42(6) (2023) 133-149.
147. N.Kansal, P.Garg and O.Singh, Temperature based topological indices and QSPR analysis for COVOD-19 drugs, *Polycyclic Aromatic Compounds*, DOI: 10.1080/10406638.2022.2086271.
148. A.E.Nabeel and N.K.Hussin, Temperature and multiplicative temperature indices of nanotubes, *Tikrit Journal of Pure Science*, 25(4) (2020) 109-116.
149. Y-F.Zhang, M.U.Ghani, F.Sultan, M.Inc and M.Cancan, Connecting SiO4 in silicate and silicate chain networks to compute Kulli temperature indices, *Molecules*, 2022, 27, 7533.
150. W.Zhen, A.Khalid, P.Ali, H.Rehman, M.K.Siddique and H.Ullah, Topological study of some COVOD-19 drugs, *Polycyclic Aromatic Compounds,* DOI: 10.1080/10406638.2022.2025864.
151. V.R.Kulli, New direction in the theory of graph index in graphs, *International Journal of Engineering Sciences & Research Technology*, 11(12) (2022) 1-8. DOI: 10.5281/zenodo.7505790.
152. V.R.Kulli, Hyper E-Banhatti indices of certain networks, *International Journal of Mathematical Archive*, 13(12) (2022) 1-10.
153. V.R.Kulli, E-BanhattiSombor indices, *International Journal of Mathematics and Computer Research,* 10(12) (2022) 2986-2994. DOI: https://doi.org/10.47191/ijmcr/v10i12.01.
154. V.R.Kulli, The (a, b)-KA E-Banhatti indices of graphs, *Journal of Mathematics and Informatics*, 23 (2022) 55-60. DOI: http://dx.doi.org/10.22457/jmi.v23a06214.
155. V.R.Kulli, Product connectivity E-Banhatti indices of certain nanotubes, *Annals of Pure and Applied Mathematics*, 27(1) (2023) 7-12. DOI: http://dx.doi.org/10.22457/apam.v27n1a02897.
156. V.R.Kulli, FE-Banhatti index of certain nanostructures, *International Journal of Mathematical Archive*, 14(1) (2023) 21-29.
157. V.R.Kulli, Atom bond connectivity E-Banhatti indices, *International Journal of Mathematics and Computer Research*, 11(1) (2023) 3201-3208. DOI: https://doi.org/10.47191/ijmcr/v11i1.13.